Chapter 33

Electromagnetic Waves

33.2: Maxwell's Rainbow:

As the figure shows, we now know a wide spectrum (or range) of electromagnetic waves: Maxwell's rainbow. In the wavelength scale in the figure, (and similarly the corresponding frequency scale), each scale marker represents a change in wavelength (and correspondingly in frequency) by a factor of 10.

The scale is open-ended; the wavelengths/frequencies of electromagnetic waves have **no inherent upper or lower bound.**
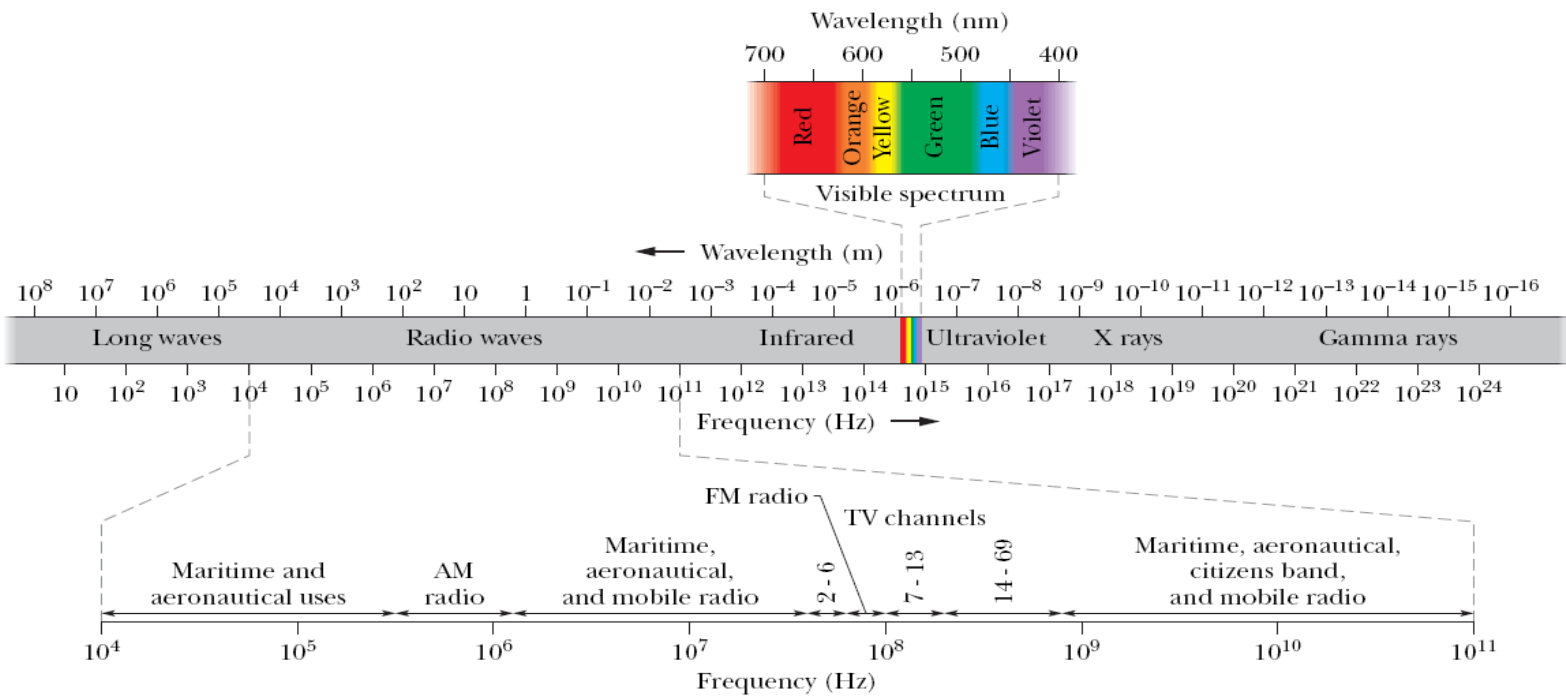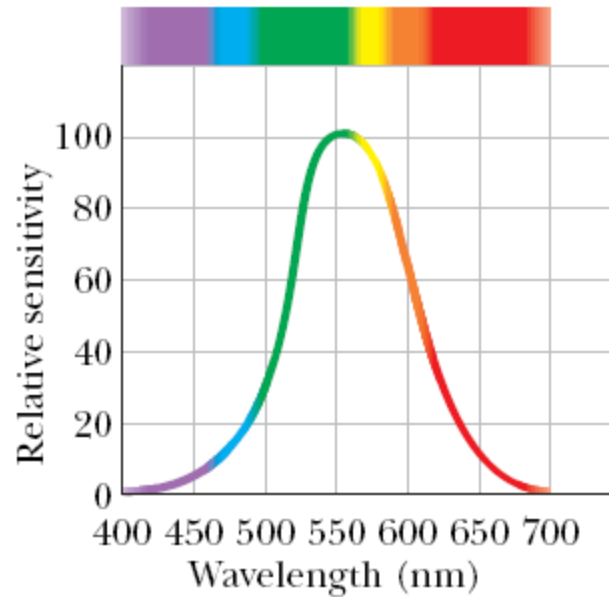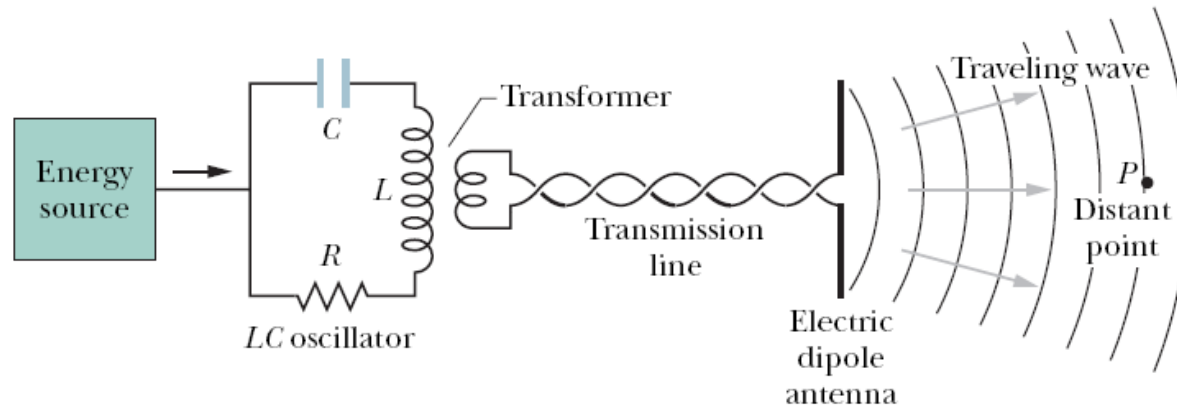

Fig. 33-1 The electromagnetic spectrum.

Fig. 33-2 The relative sensitivity of the average human eye to electromagnetic waves at different wavelengths. This portion of the electromagnetic spectrum to which the eye is sensitive is called *visible light*.

Fig. 33-3 An arrangement for generating a traveling electromagnetic wave in the shortwave radio region of the spectrum: an *LC* oscillator produces a sinusoidal current in the antenna, which generates the wave. *P* is a distant point at which a detector can monitor the wave traveling past it.

Some electromagnetic waves, including x rays, gamma rays, and visible light, are *radiated (emitted)* from sources that are of atomic or nuclear size. Figure 33-3 shows the generation of such waves. At its heart is an *LC oscillator,* which establishes an angular frequency w(=1/√(LC)). Charges and currents in this circuit vary sinusoidally at this frequency.

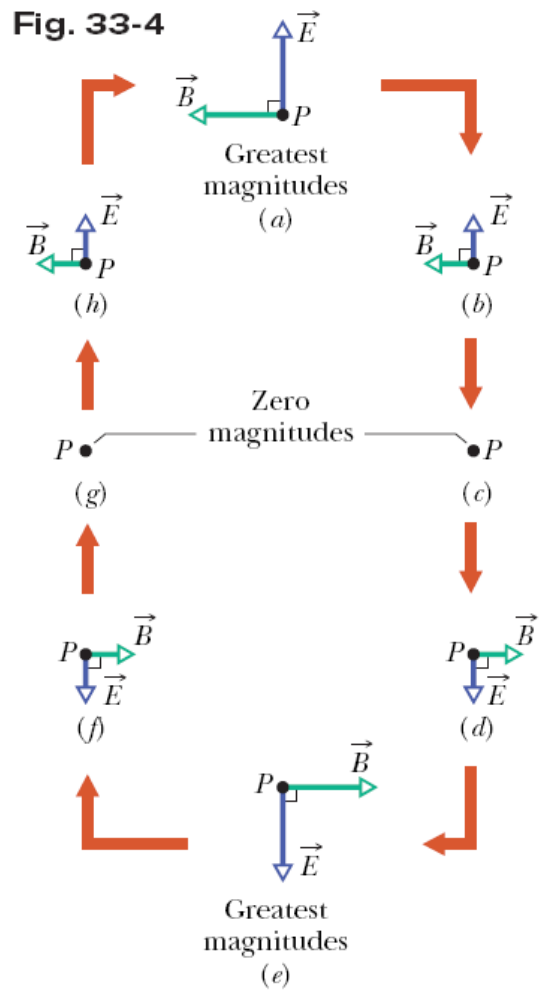# 33.3: The Traveling Wave, Qualitatively:

Fig. 33-4

Figure 33-4 shows how the electric field and the magnetic field change with time as one wavelength of the wave sweeps past the distant point P in the last figure; in each part of Fig. 33-4, the wave is traveling directly out of the page.

At a distant point, such as P, the curvature of the waves is small enough to neglect it. At such points, the wave is said to be a *plane wave*.

Here are some key features regardless of how the waves are generated:

1. The electric and magnetic fields and are always perpendicular to the direction in which the wave is traveling. The wave is a *transverse wave*.

2. The electric field is always perpendicular to the magnetic field.

3. The cross product always gives the direction in which the wave travels.

4. The fields always vary sinusoidally. The fields vary with the same frequency and are *in phase* with each other.

33.3: The Traveling Wave, Qualitatively:

We can write the electric and magnetic fields as sinusoidal functions of position x (along the path of the wave) and time t :

$$E = E_m \sin(kx - \omega t),$$

$$B = B_m \sin(kx - \omega t),$$

Here E_m and B_m are the amplitudes of the fields and, ω and k are the angular frequency and angular wave number of the wave, respectively.

All electromagnetic waves, including visible light, have the same speed c in vacuum.

The speed of the wave (in vacuum) is given by c.

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \qquad \text{(wave speed)},$$

Its value is about 3.0×10^8 m/s.

33.3: The Traveling Wave, Quantitatively:

The dashed rectangle of dimensions dx and h in Fig. 33-6 is fixed at point P on the x axis and in the xy plane.

As the electromagnetic wave moves rightward past the rectangle, the magnetic flux B through the rectangle changes and—according to Faraday's law of induction—induced electric fields appear throughout the region of the rectangle. We take $\mathbf{E}$ and $\mathbf{E} + \mathbf{dE}$ to be the induced fields along the two long sides of the rectangle. These induced electric fields are, in fact, the electrical component of the electromagnetic wave.

(a) Wavefronts — Ray

(b) $\vec{E}$ $\vec{B}$ — Electric component — Magnetic component
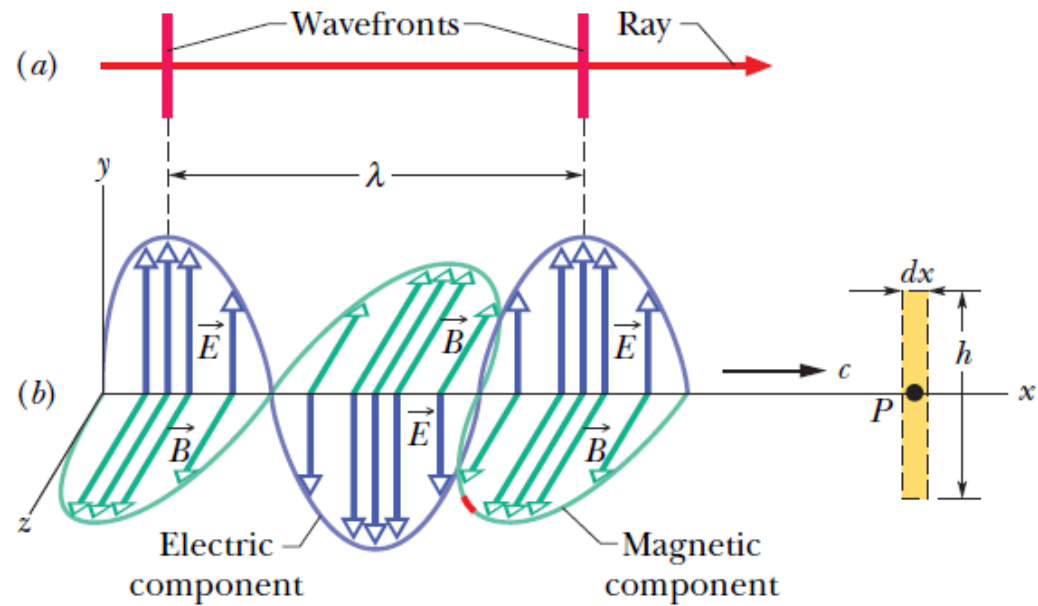
$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt} = (E + dE)h - Eh = h\,dE.$$

$$\frac{d\Phi_B}{dt} = h\,dx\,\frac{dB}{dt} \implies h\,dE = -h\,dx\,\frac{dB}{dt} \implies \frac{dE}{dx} = -\frac{dB}{dt}.$$

$$\frac{\partial E}{\partial x} = kE_m \cos(kx - \omega t)$$

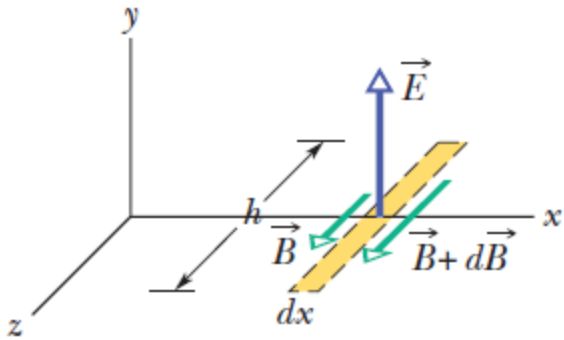$$\frac{\partial B}{\partial t} = -\omega B_m \cos(kx - \omega t).$$

$$kE_m \cos(kx - \omega t) = \omega B_m \cos(kx - \omega t).$$

$$\boxed{\frac{E_m}{B_m} = c \qquad \text{(amplitude ratio)},}$$

33.4: The Traveling Wave, Quantitatively:

The oscillating electric field induces an oscillating and perpendicular magnetic field.

Fig. 33-7 The sinusoidal variation of the electric field through this rectangle, located (but not shown) at point *P* in *Fig. 33-5b,* **E** induces magnetic fields along the rectangle. The instant shown is that of Fig. 33-6: is decreasing in magnitude, and the magnitude of the induced magnetic field is greater on the right side of the rectangle than on the left.

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt},$$

$$\oint \vec{B} \cdot d\vec{s} = -(B + dB)h + Bh = -h\,dB.$$

$$\Phi_E = (E)(h\,dx), \qquad \frac{d\Phi_E}{dt} = h\,dx\,\frac{dE}{dt}$$

$$-h\,dB = \mu_0 \varepsilon_0 \left(h\,dx\,\frac{dE}{dt} \right)$$

$$-\frac{\partial B}{\partial x} = \mu_0 \varepsilon_0 \frac{\partial E}{\partial t}.$$

$$-kB_m \cos(kx - \omega t) = -\mu_0 \varepsilon_0 \omega E_m \cos(kx - \omega t),$$

$$\frac{E_m}{B_m} = \frac{1}{\mu_0 \varepsilon_0 (\omega/k)} = \frac{1}{\mu_0 \varepsilon_0 c}.$$

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \quad \text{(wave speed),}$$

33.5: Energy Transport and the Poynting Vector:

The direction of the Poynting vector $\vec{S}$ of an electromagnetic wave at any point gives the wave's direction of travel and the direction of energy transport at that point.

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B} \qquad \text{(Poynting vector).}$$

$$S = \left(\frac{\text{energy/time}}{\text{area}} \right)_{\text{inst}} = \left(\frac{\text{power}}{\text{area}} \right)_{\text{inst}}.$$

$$S = \frac{1}{\mu_0} EB, \qquad \Longrightarrow \qquad S = \frac{1}{c\mu_0} E^2$$

$$I = S_{\text{avg}} = \left(\frac{\text{energy/time}}{\text{area}} \right)_{\text{avg}} = \left(\frac{\text{power}}{\text{area}} \right)_{\text{avg}} = \frac{1}{c\mu_0} [E^2]_{\text{avg}} = \frac{1}{c\mu_0} [E_m^2 \sin^2(kx - \omega t)]_{\text{avg}}.$$

$$E_{\text{rms}} = \frac{E_m}{\sqrt{2}}. \qquad \Longrightarrow \qquad I = \frac{1}{c\mu_0} E_{\text{rms}}^2.$$
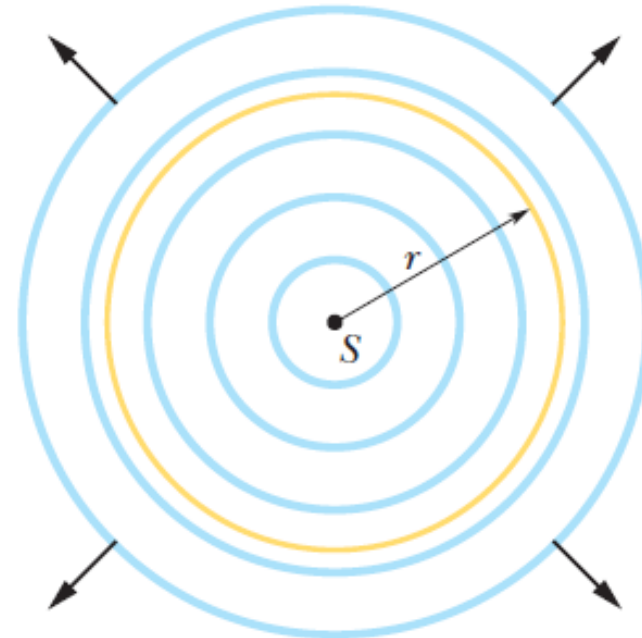
The energy density u $(= \frac{1}{2}\varepsilon_0 E^2)$ within an electric field, can be written as:

$$u_E = \tfrac{1}{2}\varepsilon_0 E^2 = \tfrac{1}{2}\varepsilon_0 (cB)^2. = \tfrac{1}{2}\varepsilon_0 \frac{1}{\mu_0 \varepsilon_0} B^2 = \frac{B^2}{2\mu_0}$$

33.5: Energy Transport and the Poynting Vector:

The energy emitted by light source S must pass through the sphere of radius r.

$$I = \frac{\text{power}}{\text{area}} = \frac{P_s}{4\pi r^2}$$

Fig. 33-8 A point source S emits electromagnetic waves uniformly in all directions. The spherical wavefronts pass through an imaginary sphere of radius r that is centered on S.

Example, Light Wave rms values of electric and magnetic fields:

When you look at the North Star (Polaris), you intercept light from a star at a distance of 431 ly and emitting energy at a rate of 2.2×10^3 times that of our Sun ($P_{sun} = 3.90 \times 10^{26}$ W). Neglecting any atmospheric absorption, find the rms values of the electric and magnetic fields when the starlight reaches you.

KEY IDEAS

1. The rms value E_{rms} of the electric field in light is related to the intensity I of the light via Eq. 33-26 ($I = E_{rms}^2/c\mu_0$).

2. Because the source is so far away and emits light with equal intensity in all directions, the intensity I at any distance r from the source is related to the source's power P_s via Eq. 33-27 ($I = P_s/4\pi r^2$).

3. The magnitudes of the electric field and magnetic field of an electromagnetic wave at any instant and at any point in the wave are related by the speed of light c according to Eq. 33-5 ($E/B = c$). Thus, the rms values of those fields are also related by Eq. 33-5.

Electric field: Putting the first two ideas together gives us

$$I = \frac{P_s}{4\pi r^2} = \frac{E_{rms}^2}{c\mu_0}$$

and

$$E_{rms} = \sqrt{\frac{P_s c\mu_0}{4\pi r^2}}.$$

Substituting $P_s = (2.2 \times 10^3)(3.90 \times 10^{26}$ W), $r = 431$ ly $= 4.08 \times 10^{18}$ m, and values for the constants, we find

$$E_{rms} = 1.24 \times 10^{-3} \text{ V/m} \approx 1.2 \text{ mV/m}. \qquad \text{(Answer)}$$

Magnetic field: From Eq. 33-5, we write

$$B_{rms} = \frac{E_{rms}}{c} = \frac{1.24 \times 10^{-3} \text{ V/m}}{3.00 \times 10^8 \text{ m/s}}$$

$$= 4.1 \times 10^{-12} \text{ T} = 4.1 \text{ pT}.$$

Cannot compare the fields: Note that E_{rms} ($= 1.2$ mV/m) is small as judged by ordinary laboratory standards, but B_{rms} ($= 4.1$ pT) is quite small. This difference helps to explain why most instruments used for the detection and measurement of electromagnetic waves are designed to respond to the electric component of the wave. It is wrong, however, to say that the electric component of an electromagnetic wave is "stronger" than the magnetic component. You cannot compare quantities that are measured in different units. However, these electric and magnetic components are on an equal basis because their average energies, which *can* be compared, are equal.

33.6: Radiation Pressure:

Electromagnetic waves have linear momentum and thus can exert a pressure on an object when shining on it.

During the interval Δt, *the object gains* an energy ΔU from the radiation. If the object is free to move and that the radiation is entirely **absorbed** (taken up) by the object, then the momentum change Δp is given by:

$$\Delta p = \frac{\Delta U}{c} \qquad \text{(total absorption)},$$

If the radiation is entirely reflected back along its original path, the magnitude of the momentum change of the object is twice that given above, or

$$\Delta p = \frac{2\,\Delta U}{c} \qquad \text{(total reflection back along path)}.$$

Since $\quad F = \dfrac{\Delta p}{\Delta t}.$ and $\quad I = \dfrac{\text{power}}{\text{area}} = \dfrac{\text{energy/time}}{\text{area}} \quad$ it follows that:
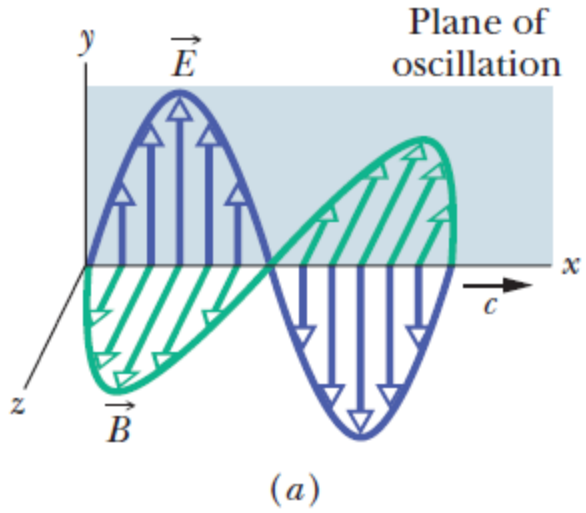
$$F = \frac{IA}{c} \qquad \text{(total absorption).}$$

AND

$$F = \frac{2IA}{c} \qquad \text{(total reflection back along path).}$$

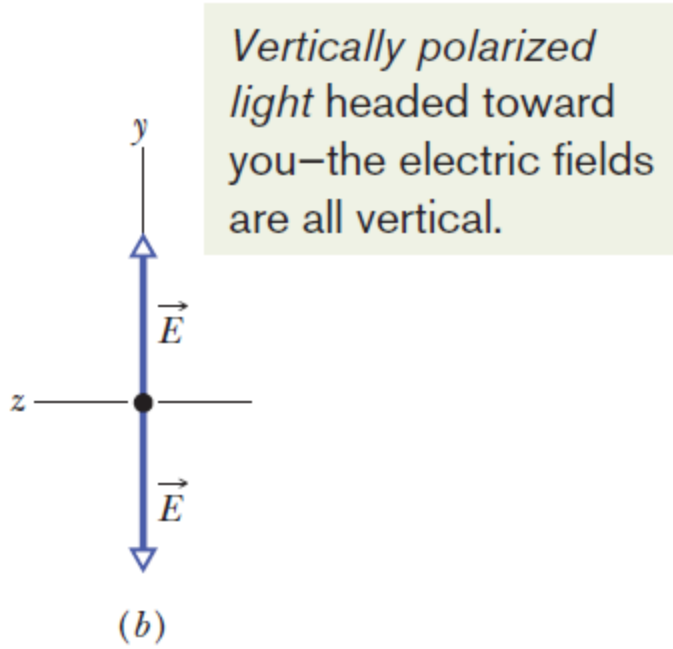Finally, the radiation pressure in the two cases are:

$$p_r = \frac{I}{c} \qquad \text{(total absorption)}$$

$$p_r = \frac{2I}{c} \qquad \text{(total reflection back along path).}$$

33.7: Polarization:

Fig. 33-9 (a) The plane of oscillation of a polarized electromagnetic wave. (b) To represent the polarization, we view the plane of oscillation head-on and indicate the directions of the oscillating electric field with a double arrow.

(a)

Vertically polarized light headed toward you–the electric fields are all vertical.

(b)

33.7: Polarization:

An electric field component parallel to the polarizing direction is passed (*transmitted*) by a polarizing sheet; a component perpendicular to it is absorbed.

Unpolarized light headed toward you—the electric fields are in all directions in the plane.

$\vec{E}$

This is a quick way to symbolize unpolarized light.

(a)

(b)
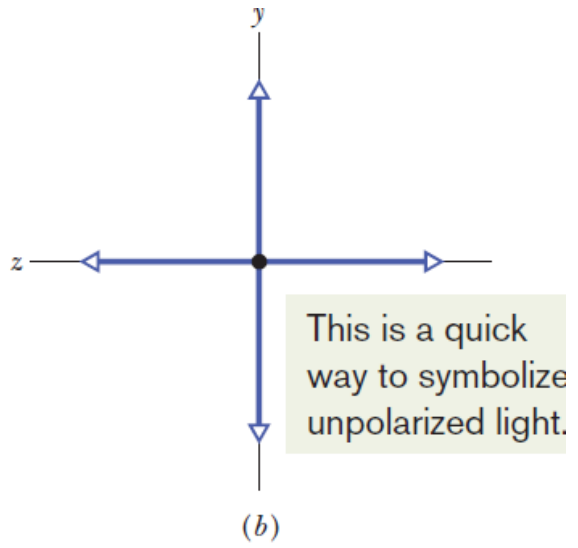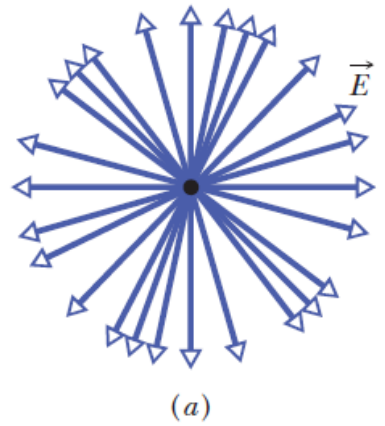
Fig. 33-10 (*a*) Unpolarized light consists of waves with randomly directed electric fields. Here the waves are all traveling along the same axis, directly out of the page, and all have the same amplitude E. (*b*) A second way of representing unpolarized light—the light is the superposition of two polarized waves whose planes of oscillation are perpendicular to each other.

If the intensity of original unpolarized light is I_o, then the intensity of the emerging light through the polarizer, I, is half of that.

$$I = \tfrac{1}{2}I_0.$$

33.7: Polarization: Intensity of Polarized Light

Suppose now that the light reaching a polarizing sheet is already polarized.

Figure 33-12 shows a polarizing sheet in the plane of the page and the electric field of such a polarized light wave traveling toward the sheet (and thus prior to any absorption).

We can resolve E into two components relative to the polarizing direction of the sheet: parallel component E_y is transmitted by the sheet, and perpendicular component E_z is absorbed. Since θ is the angle between and the polarizing direction of the sheet, the transmitted parallel component is

$$E_y = E \cos \theta.$$

Since
$$I = E_{rms}^2 / c \mu_0$$

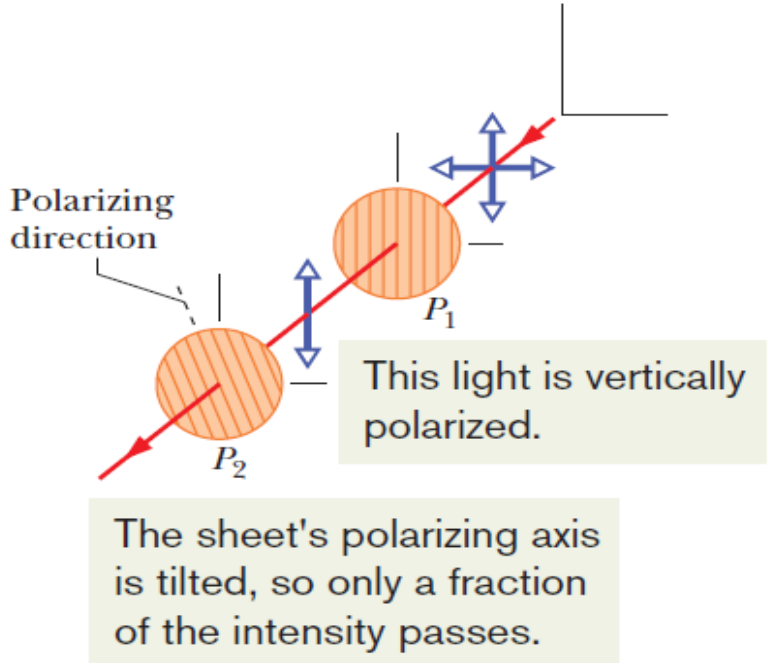$$I = I_0 \cos^2 \theta.$$

The sheet's polarizing axis is vertical, so only vertical components of the electric fields pass.
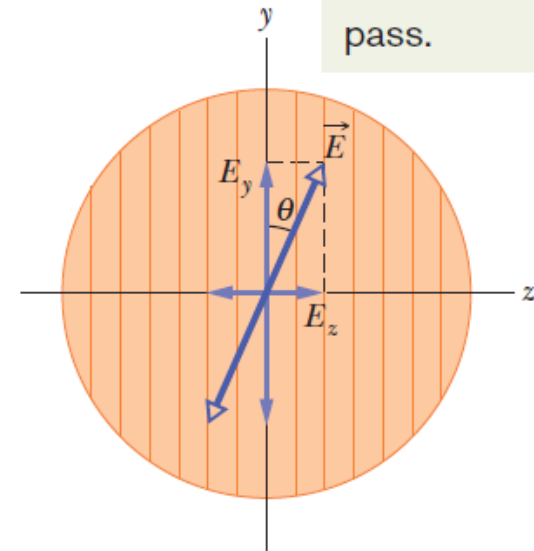
Fig. 33-12 Polarized light approaching a polarizing sheet. The electric field $\vec{E}$ of the light can be resolved into components E_y (parallel to the polarizing direction of the sheet) and E_z (perpendicular to that direction). Component E_y will be transmitted by the sheet; component E_z will be absorbed.
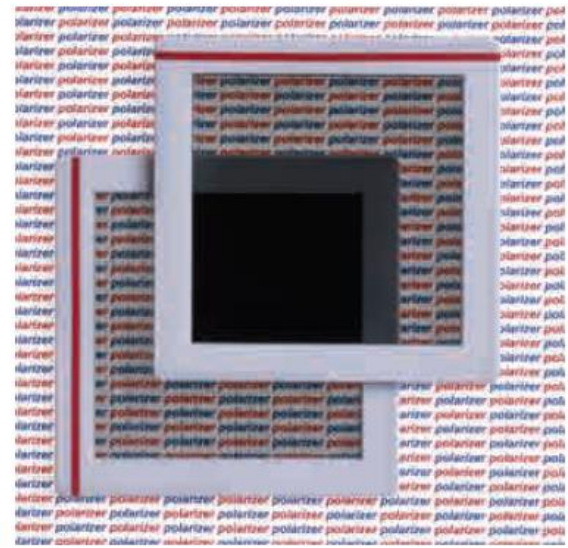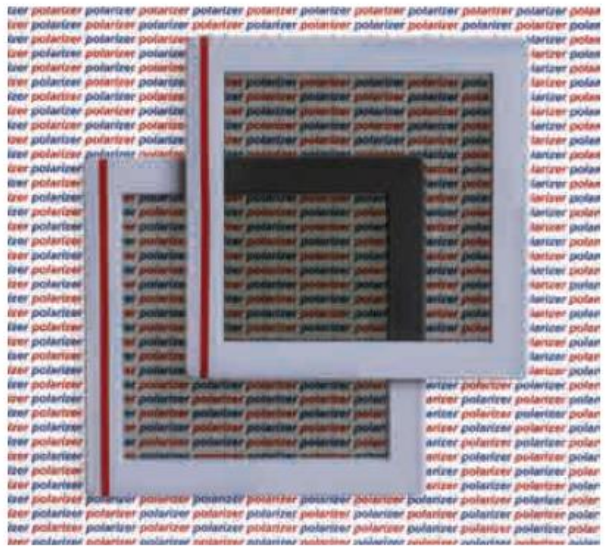
33.7: Polarization: Intensity of Polarized Light

Polarizing direction

P_1

This light is vertically polarized.

P_2

The sheet's polarizing axis is tilted, so only a fraction of the intensity passes.

Fig. 33-13 The light transmitted by polarizing sheet P_1 is vertically polarized, as represented by the vertical double arrow. The amount of that light that is then transmitted by polarizing sheet P_2 depends on the angle between the polarization direction of that light and the polarizing direction of P_2 (indicated by the lines drawn in the sheet and by the dashed line).

Fig. 33-14 (*a*) Overlapping polarizing sheets transmit light fairly well when their polarizing directions have the same orientation, but (*b*) they block most of the light when they are crossed. (*Richard Megna/Fundamental Photographs.*)

(a)

(b)

Example, Polarization and Intensity:

Figure 33-15a, drawn in perspective, shows a system of three polarizing sheets in the path of initially unpolarized light. The polarizing direction of the first sheet is parallel to the y axis, that of the second sheet is at an angle of 60° counterclockwise from the y axis, and that of the third sheet is parallel to the x axis. What fraction of the initial intensity I_0 of the light emerges from the three-sheet system, and in which direction is that emerging light polarized?
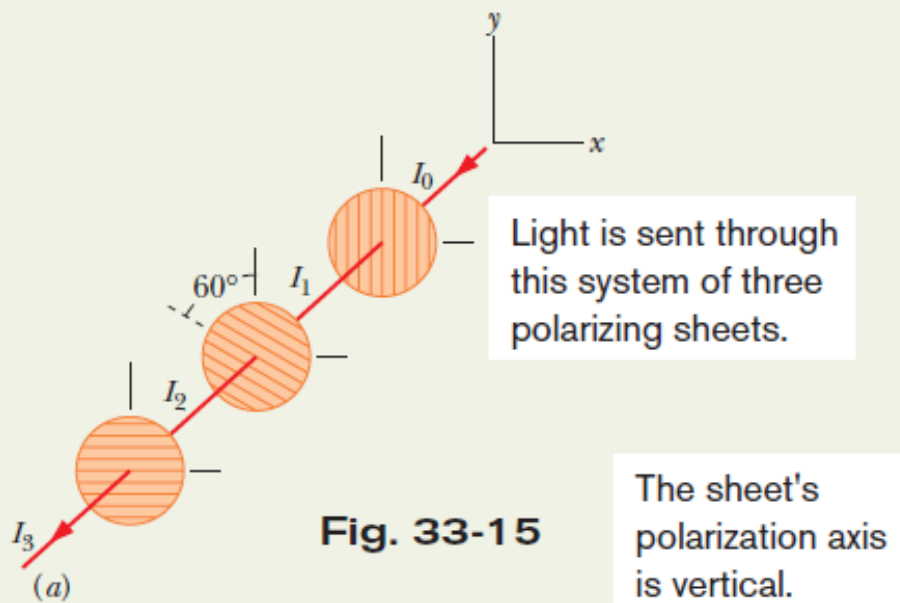

Light is sent through this system of three polarizing sheets.

Fig. 33-15

The sheet's polarization axis is vertical.

(a)

First sheet: The original light wave is represented in Fig. 33-15b, using the head-on, double-arrow representation of Fig. 33-10b. Because the light is initially unpolarized, the intensity I_1 of the light transmitted by the first sheet is given by the one-half rule (Eq. 33-36):

$$I_1 = \tfrac{1}{2} I_0.$$

Second sheet: Because the light reaching the second sheet is polarized, the intensity I_2 of the light transmitted by that sheet is given by the cosine-squared rule (Eq. 33-38). The angle θ in the rule is the angle between the polarization direction of the entering light (parallel to the y axis) and the polarizing direction of the second sheet (60° counterclockwise from the y axis), and so θ is 60°. (The larger angle between the two directions, namely 120°, can also be used.) We have

$$I_2 = I_1 \cos^2 60°.$$

The polarization of this transmitted light is parallel to the polarizing direction of the sheet transmitting it—that is, 60° counterclockwise from the y axis, as shown in the head-on view of Fig. 33-15d.

Third sheet: Because the light reaching the third sheet is polarized, the intensity I_3 of the light transmitted by that sheet is given by the cosine-squared rule. The angle θ is now the angle between the polarization direction of the entering light (Fig. 33-15d) and the polarizing direction of the third sheet (parallel to the x axis), and so $\theta = 30°$. Thus,

$$I_3 = I_2 \cos^2 30°.$$

This final transmitted light is polarized parallel to the x axis (Fig. 33-15e). We find its intensity by substituting first for I_2 and then for I_1 in the equation above:

$$I_3 = I_2 \cos^2 30° = (I_1 \cos^2 60°) \cos^2 30°$$
$$= (\tfrac{1}{2} I_0) \cos^2 60° \cos^2 30° = 0.094 I_0.$$

Thus,
$$\frac{I_3}{I_0} = 0.094. \qquad \text{(Answer)}$$

33.8: Reflection and Refraction:

$$\theta'_1 = \theta_1 \qquad \text{(reflection).}$$

$$n_2 \sin \theta_2 = n_1 \sin \theta_1 \qquad \text{(refraction).}$$

The index of refraction, n, of a medium is equal to c/v, where v is the speed of light in that medium and c is its speed in vacuum. In the refraction law, each of the symbols n_1 and n_2 is a dimensionless constant, called the **index of refraction,** that is associated with a medium involved in the refraction. The refraction law is also called **Snell's law**.
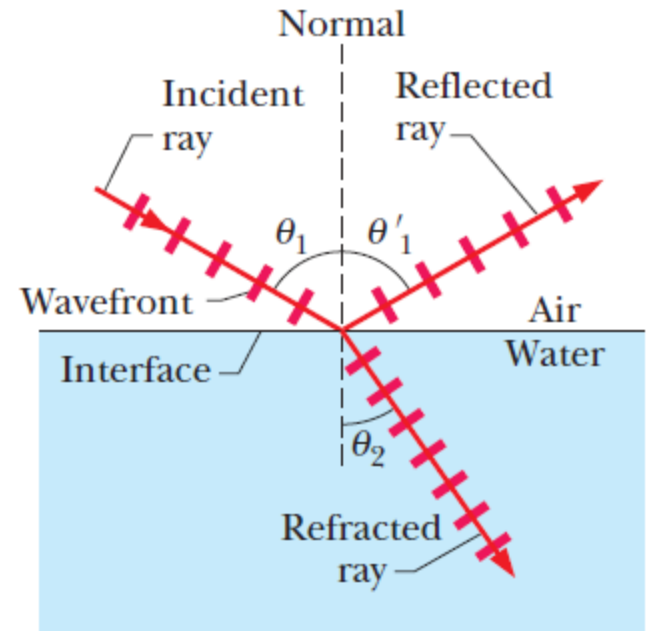
Fig. 33-16 (Continued) (b) A ray representation of (a). The angles of incidence (θ_1), reflection (θ'_1), and refraction (θ_2) are marked.

33.8: Reflection and Refraction:

	Table 33-1		
Some Indexes of Refraction[a]			
Medium	Index	Medium	Index
Vacuum	Exactly 1	Typical crown glass	1.52
Air (STP)[b]	1.00029	Sodium chloride	1.54
Water (20°C)	1.33	Polystyrene	1.55
Acetone	1.36	Carbon disulfide	1.63
Ethyl alcohol	1.36	Heavy flint glass	1.65
Sugar solution (30%)	1.38	Sapphire	1.77
Fused quartz	1.46	Heaviest flint glass	1.89
Sugar solution (80%)	1.49	Diamond	2.42

[a]For a wavelength of 589 nm (yellow sodium light).
[b]STP means "standard temperature (0°C) and pressure (1 atm)."
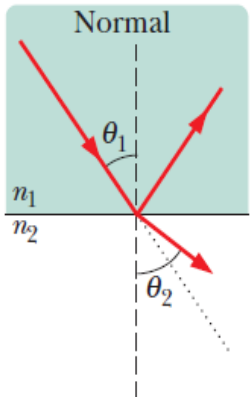
33.8: Reflection and Refraction:

Normal

n_1
n_2

θ_1

θ_2
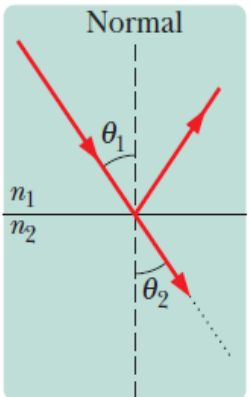
$n_2 = n_1$

(a) If the indexes match, there is no direction change.

Normal

n_1
n_2

θ_1

θ_2
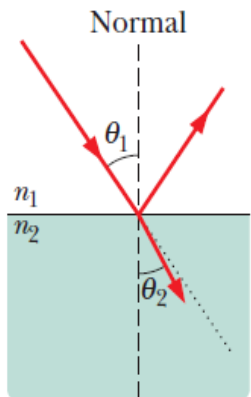
$n_2 > n_1$

(b) If the next index is greater, the ray is bent *toward* the normal.

Normal

n_1
n_2

θ_1

θ_2

$n_2 < n_1$

(c) If the next index is less, the ray is bent *away from* the normal.

Fig. 33-17 Refraction of light traveling from a medium with an index of refraction n_1 into a medium with an index of refraction n_2. (a) The beam does not bend when $n_2 = n_1$; the refracted light then travels in the *undeflected direction* (the dotted line), which is the same as the direction of the incident beam. The beam bends (b) toward the normal when $n_2 > n_1$ and (c) away from the normal when $n_2 < n_1$.
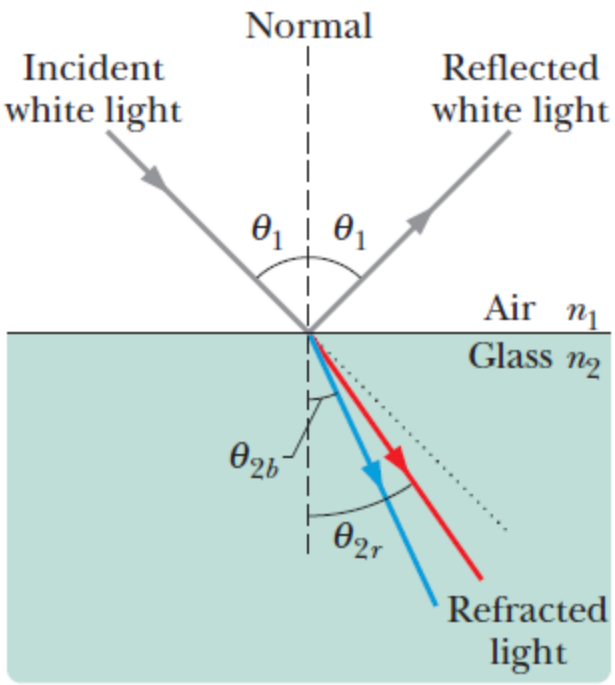
33.8: Chromatic Dispersion:
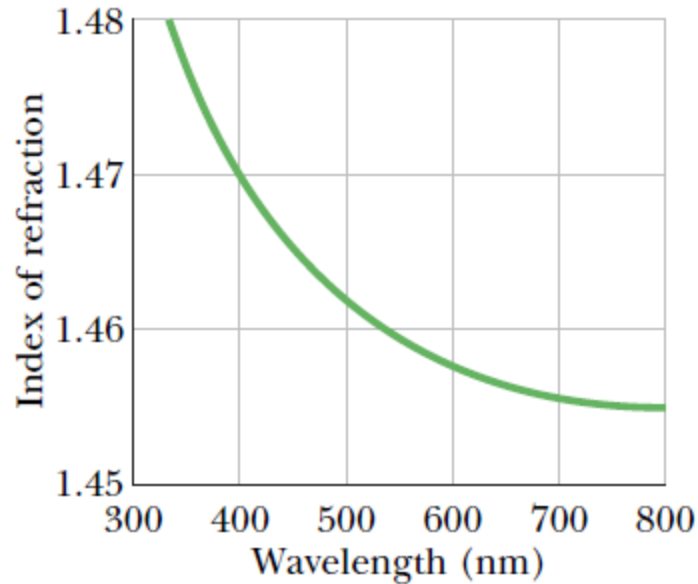
Fig. 33-18 The index of refraction as a function of wavelength for fused quartz. The graph indicates that a beam of short-wavelength light, for which the index of refraction is higher, is bent more upon entering or leaving quartz than a beam of long-wavelength light.

The index of refraction n encountered by light in any medium except vacuum depends on the wavelength of the light.
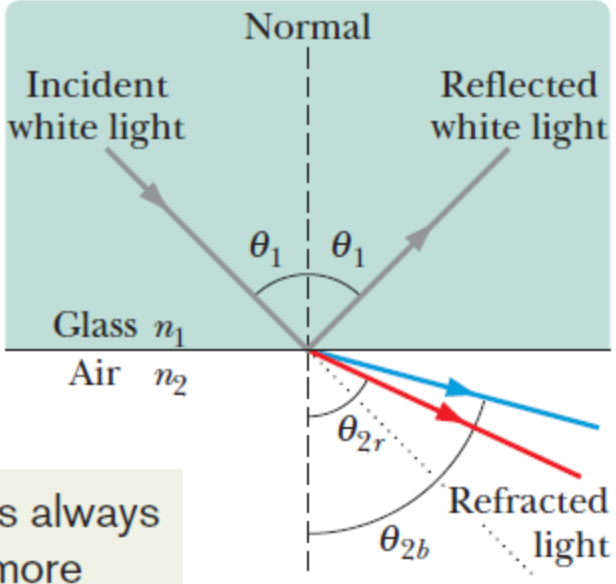
The dependence of n on wavelength implies that when a light beam consists of rays of different wavelengths, the rays will be refracted at different angles by a surface; that is, the light will be spread out by the refraction.

This spreading of light is called **chromatic dispersion**.

33.8: Chromatic Dispersion:

Fig. 33-19 Chromatic dispersion of white light. The blue component is bent more than the red component. (*a*) Passing from air to glass, the blue component ends up with the smaller angle of refraction. (*b*) Passing from glass to air, the blue component ends up with the greater angle of refraction. Each dotted line represents the direction in which the light would continue to travel if it were not bent by the refraction.

33.8: Chromatic Dispersion and Rainbow:
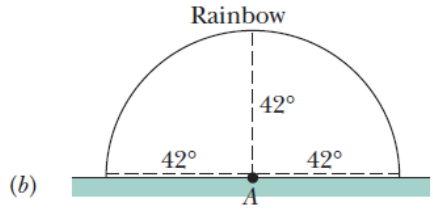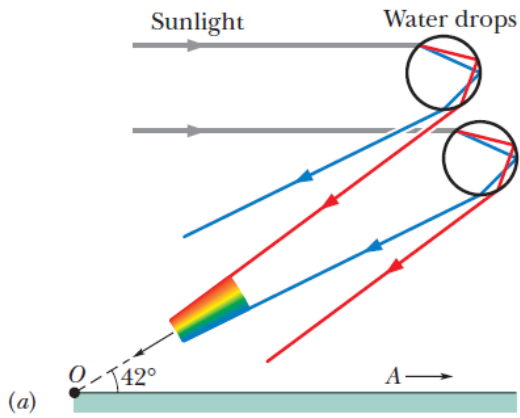
Fig. 33-21 (a) The separation of colors when sunlight refracts into and out of falling raindrops leads to a primary rainbow. The antisolar point A is on the horizon at the right. The rainbow colors appear at an angle of 42° from the direction of A. (b) Drops at 42° from A in any direction can contribute to the rainbow. (c) The rainbow arc when the Sun is higher (and thus A is lower). (d) The separation of colors leading to a secondary rainbow.
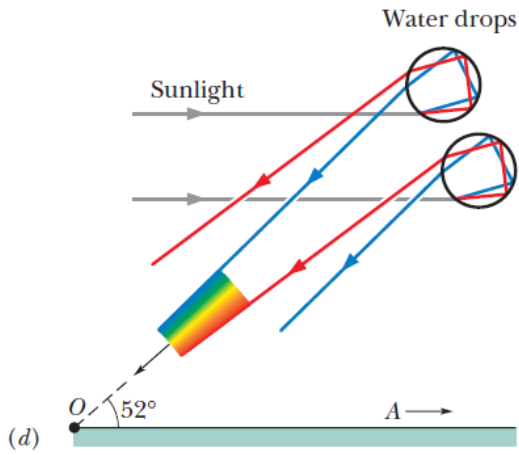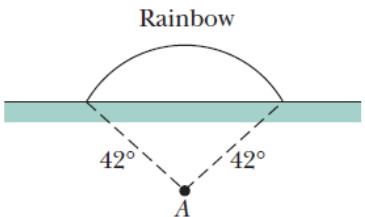
Example, Reflection and Refraction of a Monochromatic Beam:

(a) In Fig. 33-22a, a beam of monochromatic light reflects and refracts at point A on the interface between material 1 with index of refraction $n_1 = 1.33$ and material 2 with index of refraction $n_2 = 1.77$. The incident beam makes an angle of $50°$ with the interface. What is the angle of reflection at point A? What is the angle of refraction there?
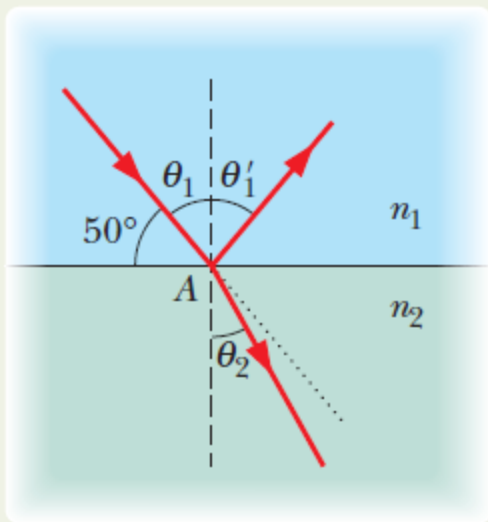

Fig. 33-22 (a)

Calculations: In Fig. 33-22a, the normal at point A is drawn as a dashed line through the point. Note that the angle of incidence θ_1 is not the given $50°$ but is $90° - 50° = 40°$. Thus, the angle of reflection is

$$\theta_1' = \theta_1 = 40°. \qquad \text{(Answer)}$$

The light that passes from material 1 into material 2 undergoes refraction at point A on the interface between the two materials. Again we measure angles between light rays and a normal, here at the point of refraction. Thus, in Fig. 33-22a, the angle of refraction is the angle marked θ_2. Solving Eq. 33-42 for θ_2 gives us

$$\theta_2 = \sin^{-1}\left(\frac{n_1}{n_2}\sin\theta_1\right) = \sin^{-1}\left(\frac{1.33}{1.77}\sin 40°\right)$$

$$= 28.88° \approx 29°. \qquad \text{(Answer)}$$

This result means that the beam swings toward the normal (it was at $40°$ to the normal and is now at $29°$). The reason is that when the light travels across the interface, it moves into a material with a greater index of refraction. *Caution:* Note that the beam does *not* swing through the normal so that it appears on the left side of Fig. 33-22a.

Example, Reflection and Refraction of a Monochromatic Beam:

Fig. 33-22

(b)
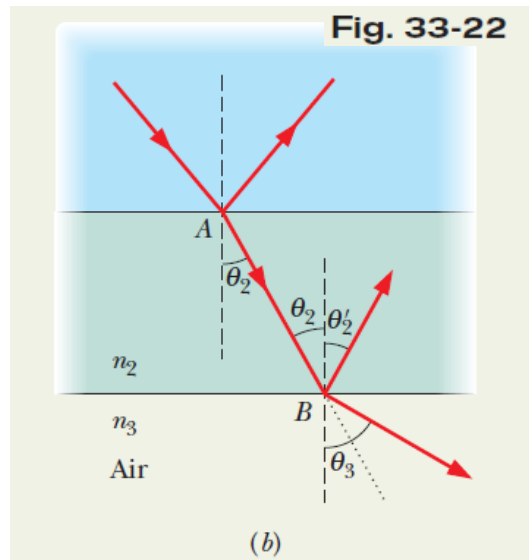
(b) The light that enters material 2 at point A then reaches point B on the interface between material 2 and material 3, which is air, as shown in Fig. 33-22b. The interface through B is parallel to that through A. At B, some of the light reflects and the rest enters the air. What is the angle of reflection? What is the angle of refraction into the air?

Calculations: We first need to relate one of the angles at point B with a known angle at point A. Because the interface through point B is parallel to that through point A, the incident angle at B must be equal to the angle of refraction θ_2, as shown in Fig. 33-22b. Then for reflection, we again use the law of reflection. Thus, the angle of reflection at B is

$$\theta_2' = \theta_2 = 28.88° \approx 29°. \qquad \text{(Answer)}$$

Next, the light that passes from material 2 into the air undergoes refraction at point B, with refraction angle θ_3. Thus, we again apply Snell's law of refraction, but this time we write Eq. 33-40 as

$$n_3 \sin \theta_3 = n_2 \sin \theta_2. \qquad (33\text{-}43)$$

Solving for θ_3 then leads to

$$\theta_3 = \sin^{-1}\left(\frac{n_2}{n_3} \sin \theta_2\right) = \sin^{-1}\left(\frac{1.77}{1.00} \sin 28.88°\right)$$

$$= 58.75° \approx 59°. \qquad \text{(Answer)}$$

This result means that the beam swings away from the normal (it was at 29° to the normal and is now at 59°). The reason is that when the light travels across the interface, it moves into a material (air) with a lower index of refraction.

33.9: Total Internal Reflection:

Fig. 33-23 (*a*) Total internal reflection of light from a point source *S* in glass occurs for all angles of incidence greater than the critical angle θ_c. At the critical angle, the refracted ray points along the air–glass interface. (*b*) A source in a tank of water. (*Ken Kay/Fundamental Photographs*)
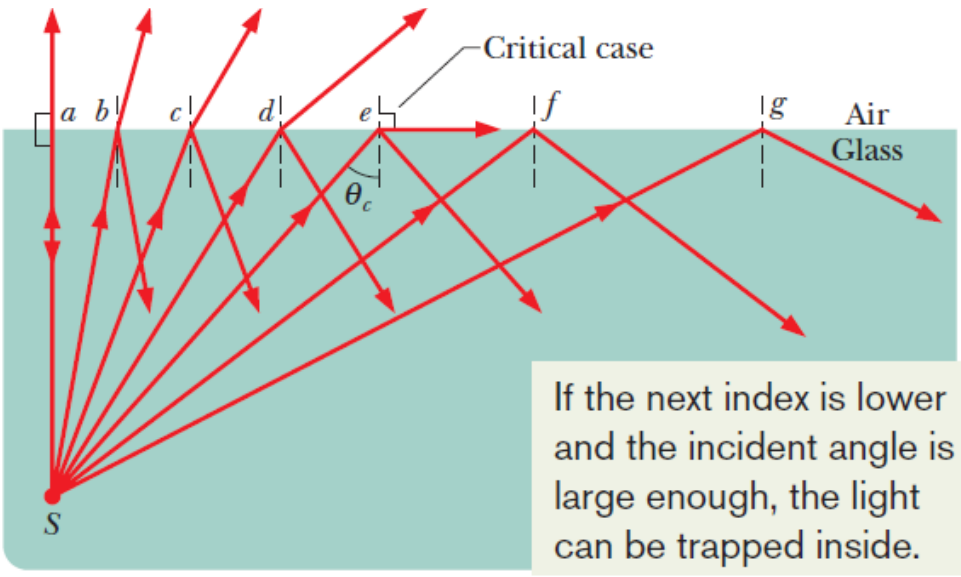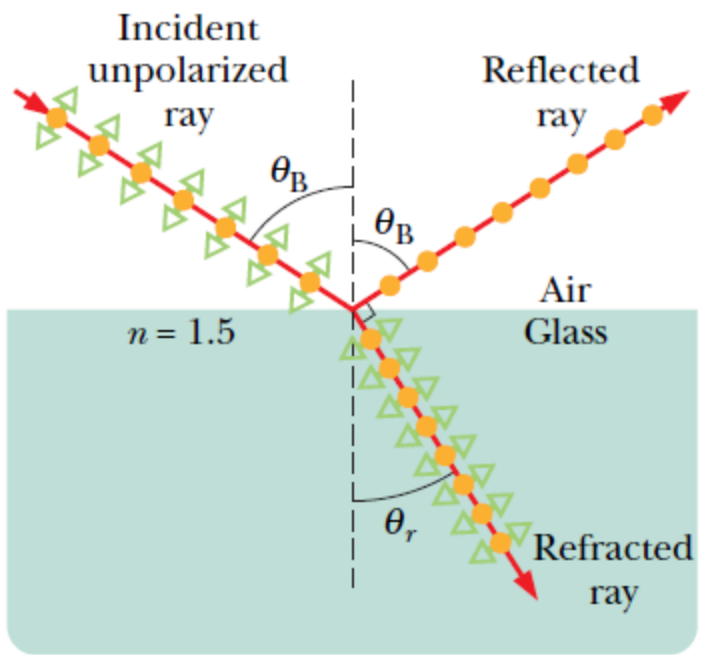

(*a*)

(*b*)

For angles of incidence larger than θ_c, such as for rays *f* and *g*, there is no refracted ray and all the light is reflected; this effect is called **total internal reflection.**

For the critical angle, $n_1 \sin \theta_c = n_2 \sin 90°,$

Which means that $\theta_c = \sin^{-1} \dfrac{n_2}{n_1}$ (critical angle).

33.10: Polarization by Reflection:

Incident unpolarized ray

Reflected ray

θ_B

θ_B

Air
Glass

$n = 1.5$

θ_r

Refracted ray

● Component perpendicular to page
◁▷ Component parallel to page

Fig. 33-25 A ray of unpolarized light in air is incident on a glass surface at the Brewster angle θ_B. The electric fields along that ray have been resolved into components perpendicular to the page (the plane of incidence, reflection, and refraction) and components parallel to the page. The reflected light consists only of components perpendicular to the page and is thus polarized in that direction. The refracted light consists of the original components parallel to the page and weaker components perpendicular to the page; this light is partially polarized.

$\theta_B + \theta_r = 90°.$

$n_1 \sin \theta_B = n_2 \sin \theta_r.$

$n_1 \sin \theta_B = n_2 \sin(90° - \theta_B) = n_2 \cos \theta_B,$

$$\theta_B = \tan^{-1} \frac{n_2}{n_1} \qquad \text{(Brewster angle).}$$