Chapter 32

Maxwell’s Equations, Magnetism of Matter

32.2: Gauss's Law for Magnetic Particles:

The simplest magnetic structure that can exist is a magnetic dipole. Magnetic monopoles do not exist (as far as we know).

$$\Phi_B = \oint \vec{B} \cdot d\vec{A} = 0 \qquad \text{(Gauss' law for magnetic fields).}$$

The law asserts that the net magnetic flux Φ_B through any closed Gaussian surface is zero. Here **B** is the magnetic field.

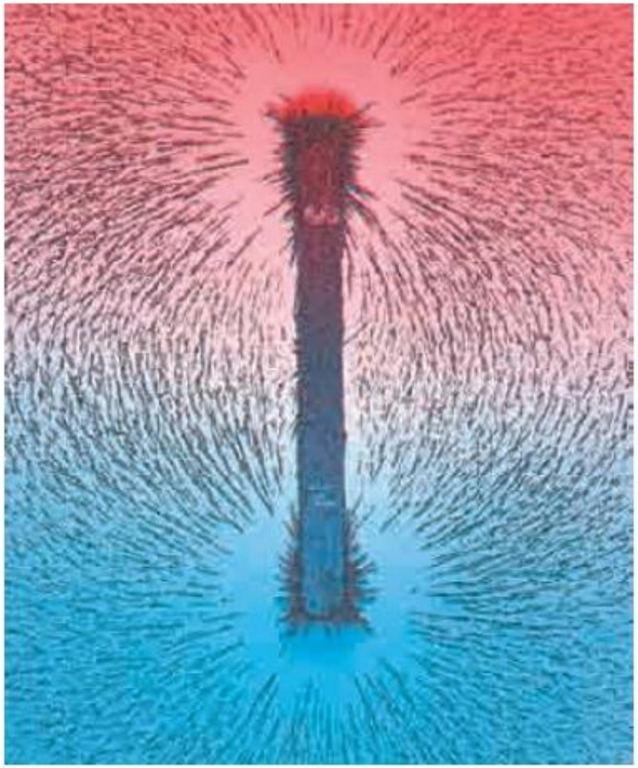


Fig. 32-2 A bar magnet is a magnetic dipole. The iron filings suggest the magnetic field lines. (Colored light fills the background.) *(Runk/Schoenberger/Grant Heilman Photography)*

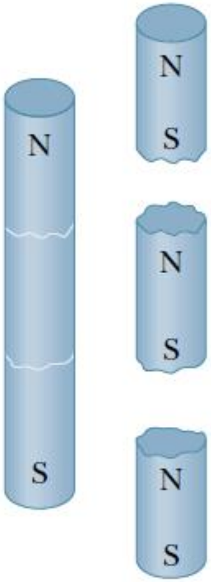


Fig. 32-3 If you break a magnet, each fragment becomes a separate magnet, with its own north and south poles.

32.2: Gauss's Law for Magnetic Particles:

$$\Phi_B = \oint \vec{B} \cdot d\vec{A} = 0 \qquad \text{(Gauss' law for magnetic fields).}$$

Gauss' law for magnetic fields holds for structures even if the Gaussian surface does not enclose the entire structure. Gaussian surface II near the bar magnet of Fig. 32-4 encloses no poles, and we can easily conclude that the net magnetic flux through it is zero.

For Gaussian surface I, it may seem to enclose only the north pole of the magnet because it encloses the label N and not the label S. However, a south pole must be associated with the lower boundary of the surface because magnetic field lines enter the surface there. Thus, Gaussian surface I encloses a magnetic dipole, and the net flux through the surface is zero.

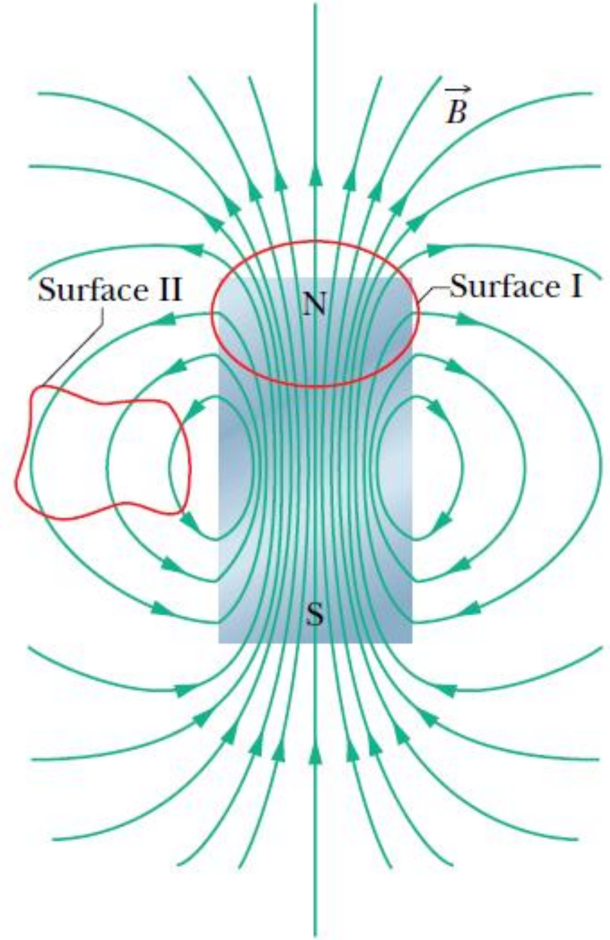


Fig. 32-4 The field lines for the magnetic field $\vec{B}$ of a short bar magnet. The red curves represent cross sections of closed, three-dimensional Gaussian surfaces.

32.3: Induced Magnetic Fields:

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} \quad \text{(Maxwell's law of induction).}$$

Here ***B*** is the magnetic field induced along a closed loop by the changing electric flux Φ_E in the region encircled by that loop.

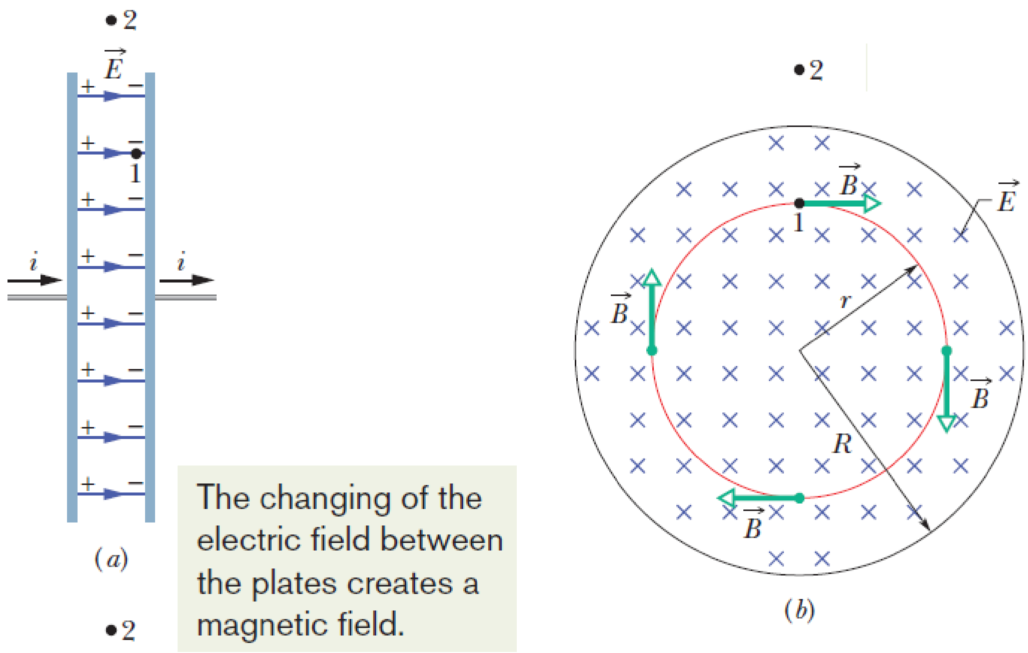


Fig. 32-5 (a) A circular parallel-plate capacitor**,** shown in side view, is being charged by a constant current *i.* (b) A view from within the capacitor, looking toward the plate at the right in *(a).*The electric field is uniform, is directed into the page (toward the plate), and grows in magnitude as the charge on the capacitor increases. The magnetic field induced by this changing electric field is shown at four points on a circle with a radius *r* less than the plate radius *R*.

32.3: Induced Magnetic Fields: Ampere Maxwell Law:

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc} \quad \text{(Ampere's law)}$$

Here i_{enc} is the current encircled by the closed loop.

In a more complete form,

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{enc} \quad \text{(Ampere–Maxwell law).}$$

When there is a current but no change in electric flux (such as with a wire carrying a constant current), the first term on the right side of the second equation is zero, and so it reduces to the first equation, Ampere's law.

The induced $\vec{E}$ direction here is opposite the induced $\vec{B}$ direction in the preceding figure.

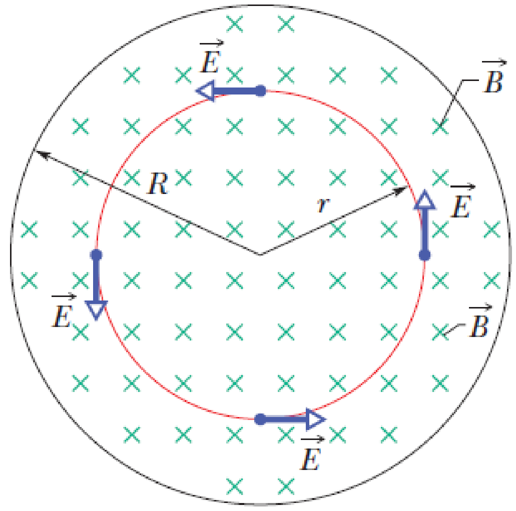
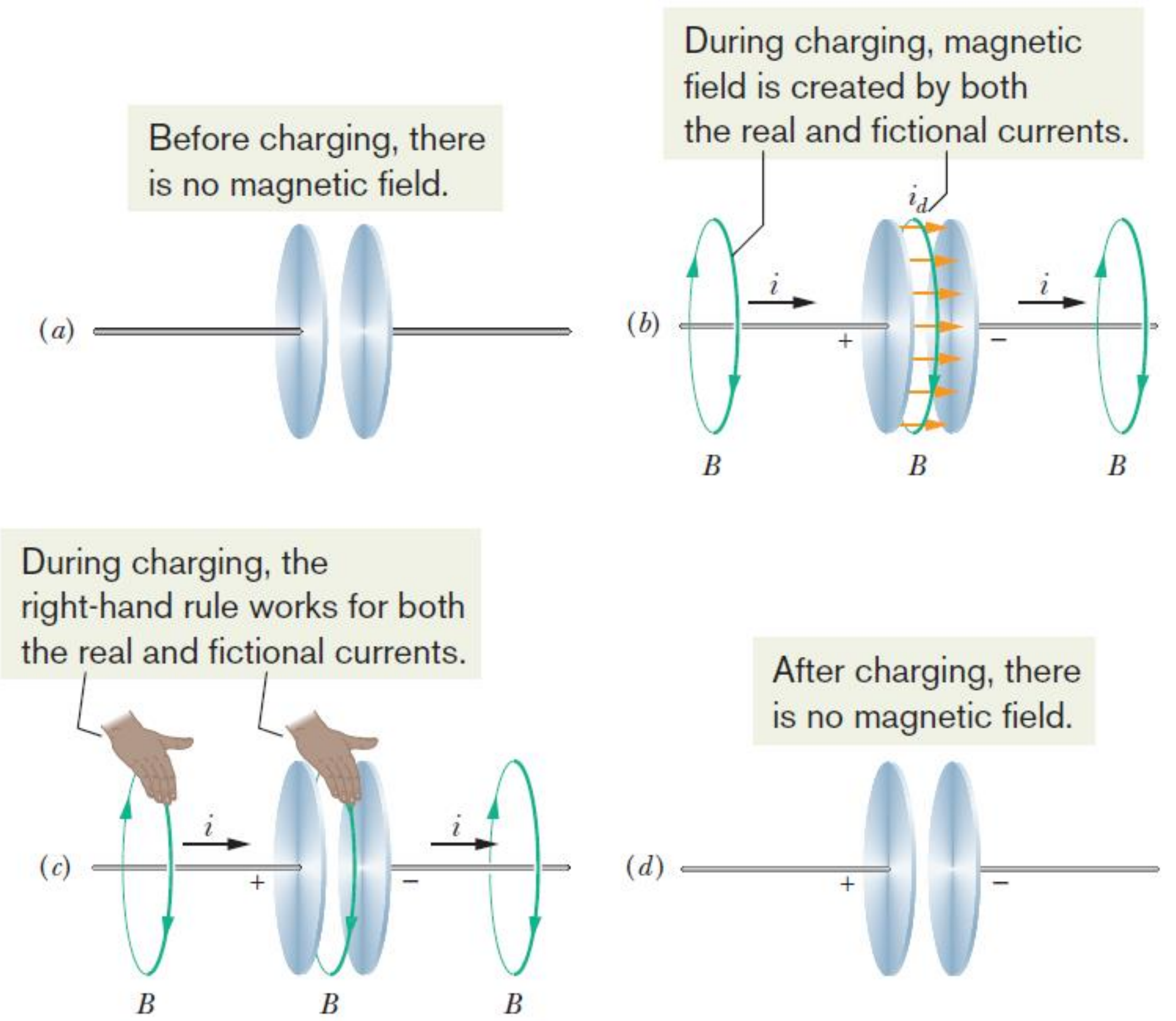


Fig. 32-6 A uniform magnetic field $\vec{B}$ in a circular region. The field, directed into the page, is increasing in magnitude. The electric field $\vec{E}$ induced by the changing magnetic field is shown at four points on a circle concentric with the circular region.

32.4: Displacement Current:

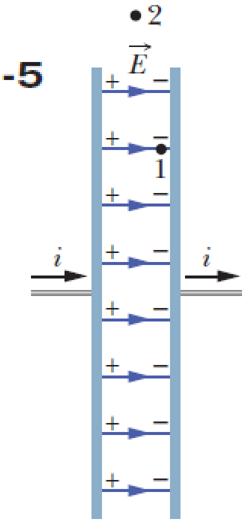
Fig. 32-7 (*a*) Before and (*d*) aft the plates are charged, there is no magnetic field. (*b*) During the char magnetic field is created by both tl current and the (fictional) displace current. (*c*) The same right-hand ru works for both currents to give the tion of the magnetic field.



Example, Magnetic Field Induced by Changing Electric Field:

Fig. 32-5

(*a*)



A parallel-plate capacitor with circular plates of radius R is being charged as in Fig. 32-5*a*.

(a) Derive an expression for the magnetic field at radius r for the case $r \le R$.

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}. \qquad (32\text{-}6)$$

We shall separately evaluate the left and right sides of this equation.

Left side of Eq. 32-6: We choose a circular Amperian loop with a radius $r \le R$ as shown in Fig. 32-5*b* because we want to evaluate the magnetic field for $r \le R$—that is, inside the capacitor. The magnetic field $\vec{B}$ at all points along the loop is tangent to the loop, as is the path element $d\vec{s}$. Thus, $\vec{B}$ and $d\vec{s}$ are either parallel or antiparallel at each point of the loop. For simplicity, assume they are parallel (the choice does not alter our outcome here). Then

$$\oint \vec{B} \cdot d\vec{s} = \oint B\, ds \cos 0° = \oint B\, ds.$$

Due to the circular symmetry of the plates, we can also assume that $\vec{B}$ has the same magnitude at every point around the loop. Thus, B can be taken outside the integral on the right side of the above equation. The integral that remains is $\oint ds$, which simply gives the circumference $2\pi r$ of the loop. The left side of Eq. 32-6 is then $(B)(2\pi r)$.

Right side of Eq. 32-6: We assume that the electric field $\vec{E}$ is uniform between the capacitor plates and directed perpendicular to the plates. Then the electric flux Φ_E through the Amperian loop is EA, where A is the area encircled by the loop within the electric field. Thus, the right side of Eq. 32-6 is $\mu_0 \varepsilon_0\, d(EA)/dt$.

Combining results: Substituting our results for the left and right sides into Eq. 32-6, we get

$$(B)(2\pi r) = \mu_0 \varepsilon_0 \frac{d(EA)}{dt}.$$

Because A is a constant, we write $d(EA)$ as $A\, dE$; so we have

$$(B)(2\pi r) = \mu_0 \varepsilon_0 A \frac{dE}{dt}. \qquad (32\text{-}7)$$

The area A that is encircled by the Amperian loop within the electric field is the *full* area πr^2 of the loop because the loop's radius r is less than (or equal to) the plate radius R. Substituting πr^2 for A in Eq. 32-7 leads to, for $r \le R$,

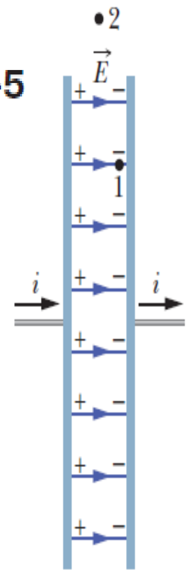
$$B = \frac{\mu_0 \varepsilon_0 r}{2} \frac{dE}{dt}. \qquad \text{(Answer)} \quad (32\text{-}8)$$

Example, Magnetic Field Induced by Changing Electric Field, cont.:

A parallel-plate capacitor with circular plates of radius R is being charged as in Fig. 32-5*a*.

Fig. 32-5

(*a*)



(*a*)

(b) Evaluate the field magnitude B for $r = R/5 = 11.0$ mm and $dE/dt = 1.50 \times 10^{12}$ V/m · s.

Calculation: From the answer to (a), we have

$$B = \frac{1}{2}\mu_0\varepsilon_0 r \frac{dE}{dt}$$
$$= \tfrac{1}{2}(4\pi \times 10^{-7}\ \text{T} \cdot \text{m/A})(8.85 \times 10^{-12}\ \text{C}^2/\text{N} \cdot \text{m}^2)$$
$$\times (11.0 \times 10^{-3}\ \text{m})(1.50 \times 10^{12}\ \text{V/m} \cdot \text{s})$$
$$= 9.18 \times 10^{-8}\ \text{T}. \qquad \text{(Answer)}$$

(c) Derive an expression for the induced magnetic field for the case $r \geq R$.

Calculation: Our procedure is the same as in (a) except we now use an Amperian loop with a radius r that is greater than the plate radius R, to evaluate B outside the capacitor. Evaluating the left and right sides of Eq. 32-6 again leads to Eq. 32-7. However, we then need this subtle point: The electric field exists only between the plates, not outside the plates. Thus, the area A that is encircled by the Amperian loop in the electric field is *not* the full area πr^2 of the loop. Rather, A is only the plate area πR^2.

Substituting πR^2 for A in Eq. 32-7 and solving the result for B give us, for $r \geq R$,

$$B = \frac{\mu_0 \varepsilon_0 R^2}{2r}\frac{dE}{dt}. \qquad \text{(Answer)} \qquad (32\text{-}9)$$

32.4: Displacement Current:

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{\text{enc}} \quad \text{(Ampere–Maxwell law).}$$

Comparing the last two terms on the right side of the above equation shows that the term $\varepsilon_0(d\Phi_E/dt)$ must have the dimension of a current. This product is usually treated as being a fictitious current called the **displacement current i_d:**

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{d,\text{enc}} + \mu_0 i_{\text{enc}} \quad \text{(Ampere–Maxwell law),}$$

in which $i_{d,enc}$ is the displacement current that is encircled by the integration loop.

The charge q on the plates of a parallel plate capacitor at any time is related to the magnitude E of the field between the plates at that time by $q = \varepsilon_0 AE,$ in which A is the plate area.

$$\frac{dq}{dt} = i = \varepsilon_0 A \frac{dE}{dt}.$$

$$i_d = \varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0 \frac{d(EA)}{dt} = \varepsilon_0 A \frac{dE}{dt}.$$

$$i_d = i \quad \text{(displacement current in a capacitor).}$$

The associated magnetic field are:

$$B = \left(\frac{\mu_0 i_d}{2\pi R^2}\right) r \quad \text{(inside a circular capacitor).}$$

AND

$$B = \frac{\mu_0 i_d}{2\pi r} \quad \text{(outside a circular capacitor).}$$

Example, Treating a Changing Electric Field as a Displacement Current:

A circular parallel-plate capacitor with plate radius R is being charged with a current i.

(a) Between the plates, what is the magnitude of $\oint \vec{B} \cdot d\vec{s}$, in terms of μ_0 and i, at a radius $r = R/5$ from their center?

Calculations: Because we want to evaluate $\oint \vec{B} \cdot d\vec{s}$ at radius $r = R/5$ (within the capacitor), the integration loop encircles only a portion $i_{d,\text{enc}}$ of the total displacement current i_d. Let's assume that i_d is uniformly spread over the full plate area. Then the portion of the displacement current encircled by the loop is proportional to the area encircled by the loop:

$$\frac{\begin{pmatrix}\text{encircled displacement} \\ \text{current } i_{d,\text{enc}}\end{pmatrix}}{\begin{pmatrix}\text{total displacement} \\ \text{current } i_d\end{pmatrix}} = \frac{\text{encircled area } \pi r^2}{\text{full plate area } \pi R^2}.$$

This gives us

$$i_{d,\text{enc}} = i_d \frac{\pi r^2}{\pi R^2}.$$

Substituting this into Eq. 32-18, we obtain

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_d \frac{\pi r^2}{\pi R^2}. \qquad (32\text{-}19)$$

Now substituting $i_d = i$ (from Eq. 32-15) and $r = R/5$ into Eq. 32-19 leads to

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i \frac{(R/5)^2}{R^2} = \frac{\mu_0 i}{25}. \qquad \text{(Answer)}$$

(b) In terms of the maximum induced magnetic field, what is the magnitude of the magnetic field induced at $r = R/5$, inside the capacitor?

Calculations: At $r = R/5$, Eq. 32-16 yields

$$B = \left(\frac{\mu_0 i_d}{2\pi R^2}\right) r = \frac{\mu_0 i_d (R/5)}{2\pi R^2} = \frac{\mu_0 i_d}{10\pi R}. \qquad (32\text{-}20)$$

$$B_{\max} = \left(\frac{\mu_0 i_d}{2\pi R^2}\right) R = \frac{\mu_0 i_d}{2\pi R}. \qquad (32\text{-}21)$$

Dividing Eq. 32-20 by Eq. 32-21 and rearranging the result, we find that the field magnitude at $r = R/5$ is

$$B = \tfrac{1}{5} B_{\max}. \qquad \text{(Answer)}$$

We should be able to obtain this result with a little reasoning and less work. Equation 32-16 tells us that inside the capacitor, B increases linearly with r. Therefore, a point $\frac{1}{5}$ the distance out to the full radius R of the plates, where $B_{\max}$ occurs, should have a field B that is $\frac{1}{5} B_{\max}$.

32.5: Maxwell's Equations:

Table 32-1

Maxwell's Equations[a]

Name	Equation	
Gauss' law for electricity	$\oint \vec{E} \cdot d\vec{A} = q_{\text{enc}}/\varepsilon_0$	Relates net electric flux to net enclosed electric charge
Gauss' law for magnetism	$\oint \vec{B} \cdot d\vec{A} = 0$	Relates net magnetic flux to net enclosed magnetic charge
Faraday's law	$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$	Relates induced electric field to changing magnetic flux
Ampere–Maxwell law	$\oint \vec{B} \cdot d\vec{s} = \mu_0\varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{\text{enc}}$	Relates induced magnetic field to changing electric flux and to current

[a]Written on the assumption that no dielectric or magnetic materials are present.

32.6: Magnets: The Magnetism of Earth:

For Earth, the south pole of the dipole is actually in the north.

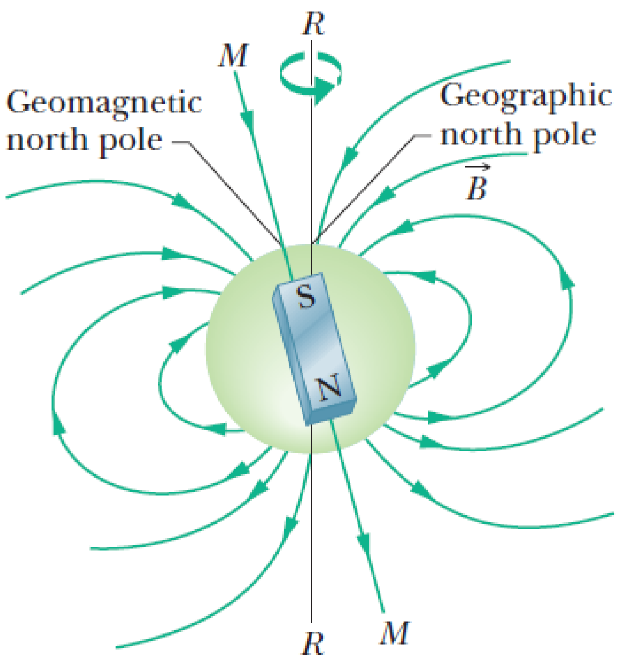


Fig. 32-8 Earth's magnetic field represented as a dipole field. The dipole axis MM makes an angle of 11.5° with Earth's rotational axis RR. The south pole of the dipole is in Earth's Northern Hemisphere.

Because Earth's magnetic field is that of a magnetic dipole, a magnetic dipole moment $\boldsymbol{\mu}$ is associated with the field.

The **field declination is the angle** (left or right) between geographic north (which is toward 90° latitude) and the horizontal component of the field.

The **field inclination is the angle** (up or down) between a horizontal plane and the field's direction.

Magnetometers measure these angles and determine the field with much precision. One can do reasonably well with just a *compass* and a *dip meter.*

The point where the field is perpendicular to Earth's surface and inward is not located at the geomagnetic north pole off Greenland as expected; instead, this so-called *dip north pole* is located in the Queen Elizabeth Islands in northern Canada, far from Greenland.

32.7: Magnetism and Electrons: Spin Magnetic Dipole Moment:

An electron has an intrinsic angular momentum called its **spin angular momentum** (or just **spin**), **S**; associated with this spin is an intrinsic spin magnetic dipole moment, $\boldsymbol{\mu}_s$. (By intrinsic, we mean that **S** and $\boldsymbol{\mu}_s$ are basic characteristics of an electron, like its mass and electric charge.)

$$\vec{\mu}_s = -\frac{e}{m}\vec{S},$$

in which *e* is the elementary charge (1.60 x10^{-19} C) and *m* is the mass of an electron (9.11 10^{31} kg).

1. Spin $\vec{S}$ itself cannot be measured. However, its component along any axis can be measured.
2. A measured component of $\vec{S}$ is *quantized,* which is a general term that means it is restricted to certain values. A measured component of $\vec{S}$ can have only two values, which differ only in sign.

Let us assume that the component of spin $\vec{S}$ is measured along the z axis of a coordinate system. Then the measured component S_z can have only the two values given by

$$S_z = m_s \frac{h}{2\pi}, \qquad \text{for } m_s = \pm\tfrac{1}{2}, \tag{32-23}$$

where m_s is called the *spin magnetic quantum number* and h (= 6.63×10^{-34} J · s) is the Planck constant, the ubiquitous constant of quantum physics.

32.7: Magnetism and Electrons: Spin Magnetic Dipole Moment:

$$\vec{\mu}_s = -\frac{e}{m}\vec{S},$$

For an electron, the spin is opposite the magnetic dipole moment.

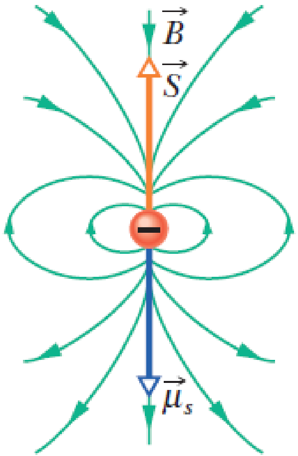


Fig. 32-10 The spin $\vec{S}$, spin magnetic dipole moment $\vec{\mu}_s$, and magnetic dipole field $\vec{B}$ of an electron represented as a microscopic sphere.

$$\Rightarrow \quad \mu_{s,z} = -\frac{e}{m}S_z. \quad \Rightarrow \quad \mu_{s,z} = \pm\frac{eh}{4\pi m},$$

$$\Rightarrow \quad \mu_B = \frac{eh}{4\pi m} = 9.27 \times 10^{-24}\text{ J/T} \qquad \text{(Bohr magneton).}$$

The orientation energy for the electron, when Bext is the exterior magnetic field aligned along the z-axis.

$$U = -\vec{\mu}_s \cdot \vec{B}_{ext} = -\mu_{s,z}B_{ext},$$

32.7: Magnetism and Electrons: Orbital Magnetic Dipole Moment:

When it is in an atom, an electron has an additional angular momentum called its orbital angular momentum, $\boldsymbol{L}_{orb}$ **.** Associated with it is an orbital magnetic dipole moment, $\boldsymbol{\mu}_{orb}$; the two are related by

$$\vec{\mu}_{\text{orb}} = -\frac{e}{2m}\vec{L}_{\text{orb}}.$$

Only the component along any axis of the orbital angular momentum can be measured, and that component is quantized

$$L_{\text{orb},z} = m_\ell \frac{h}{2\pi}, \qquad \text{for } m_\ell = 0, \pm 1, \pm 2, \ldots, \pm(\text{limit}),$$

in which is m_l called the orbital magnetic quantum number and "limit" refers to its largest allowed integer value.

Similarly, only the component of the magnetic dipole moment of an electron along an axis can be measured, and that component is quantized.

$$\mu_{\text{orb},z} = -m_\ell \frac{eh}{4\pi m} = -m_\ell \mu_B.$$

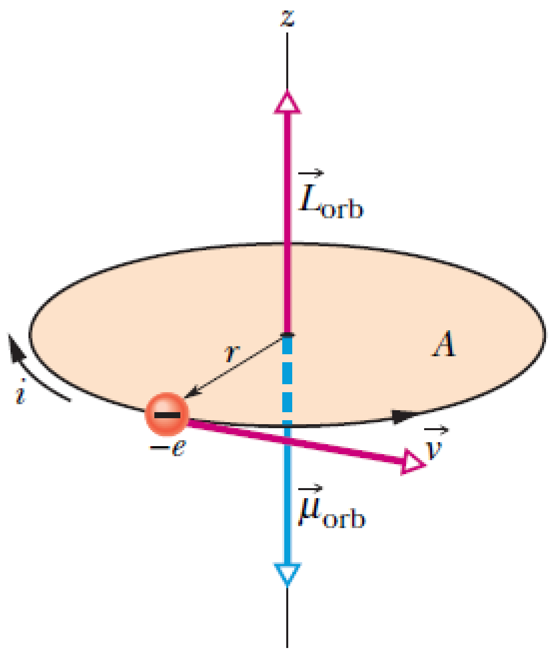
The orientation energy is:

$$U = -\vec{\mu}_{\text{orb}} \cdot \vec{B}_{\text{ext}} = -\mu_{\text{orb},z} B_{\text{ext}},$$

where the z axis is taken in the direction of $\boldsymbol{B}_{ext}$.

32.7: Magnetism and Electrons: Loop Model for Electron Orbits:

Fig. 32-11 An electron moving at constant speed v in a circular path of radius r that encloses an area A. The electron has an orbital angular momentum $\vec{L}_{orb}$ and an associated orbital magnetic dipole moment $\vec{\mu}_{orb}$. A clockwise current i (of positive charge) is equivalent to the counterclockwise circulation of the negatively charged electron.



The magnitude of the orbital magnetic dipole moment of the current loop shown is:

$$\mu_{orb} = iA,$$

Here A is the area enclosed by the loop. Since $i = \frac{\text{charge}}{\text{time}} = \frac{e}{2\pi r/v}$.

$$\mu_{orb} = \frac{e}{2\pi r/v}\pi r^2 = \frac{evr}{2}.$$

And since $L_{orb} = mrv \sin 90° = mrv.$

Therefore,

$$\vec{\mu}_{orb} = -\frac{e}{2m}\vec{L}_{orb},$$

32.7: Magnetism and Electrons:
Loop Model for Electron Orbits in a Nonuniform Field:

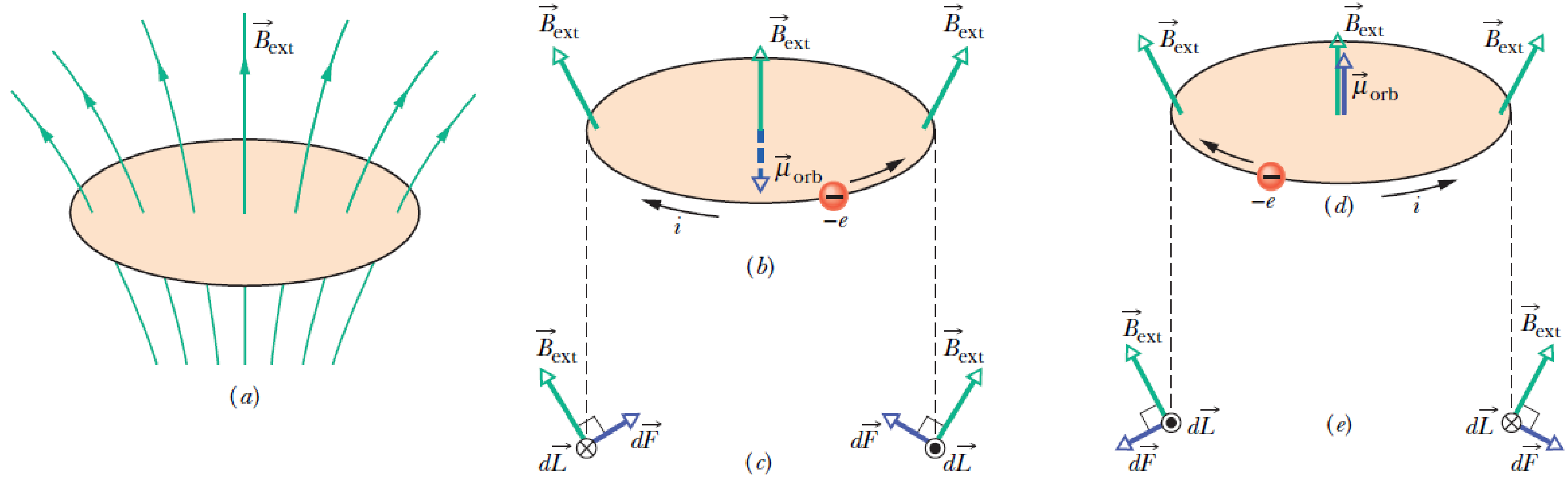


Fig. 32-12 (a) A loop model for an electron orbiting in an atom while in a nonuniform magnetic field $\mathbf{B}_{ext}$. (b) Charge e moves counterclockwise; the associated conventional current *i* is clockwise. (c) The magnetic forces ***dF*** on the left and right sides of the loop, as seen from the plane of the loop. The net force on the loop is upward. (d) Charge e now moves clockwise. (e) The net force on the loop is now downward.

$$d\vec{F} = i\, d\vec{L} \times \vec{B}_{ext}.$$

32.8: Magnetic Materials:

Each electron in an atom has an orbital magnetic dipole moment and a spin magnetic dipole moment. The resultant of these two vectors combines with similar resultants for all other electrons in the atom, and the resultant for each atom combines with those for all the other atoms in a sample of a material. In a magnetic material the combination of all these magnetic dipole moments produces a magnetic field. There are three general types of magnetism.

1. Diamagnetism: In diamagnetism, weak magnetic dipole moments are produced in the atoms of the material when the material is placed in an external magnetic field $\mathbf{B}_{ext}$; the combination gives the material as a whole only a feeble net magnetic field.

2. Paramagnetism: Each atom of such a material has a permanent resultant magnetic dipole moment, but the moments are randomly oriented in the material and the material lacks a net magnetic field. An external magnetic field $\mathbf{B}_{ext}$ can partially align the atomic magnetic dipole moments to give the material a net magnetic field.

3. Ferromagnetism: Some of the electrons in these materials have their resultant magnetic dipole moments aligned, which produces regions with strong magnetic dipole moments. An external field $\mathbf{B}_{ext}$ can align the magnetic moments of such regions, producing a strong magnetic field for the material.

32.9: Diamagnetism:

A diamagnetic material placed in an external magnetic field $\vec{B}_{ext}$ develops a magnetic dipole moment directed opposite $\vec{B}_{ext}$. If the field is nonuniform, the diamagnetic material is repelled *from* a region of greater magnetic field *toward* a region of lesser field.

If a magnetic field is applied, the diamagnetic material develops a magnetic dipole moment and experiences a magnetic force. When the field is removed, both the dipole moment and the force disappear.

Fig. 32-13 An overhead view of a frog that is being levitated in a magnetic field produced by current in a vertical solenoid below the frog. *(Courtesy A. K. Gein, High Field Magnet Laboratory, University of Nijmegen, The Netherlands)*

32.10: Paramagnetism:

A paramagnetic material placed in an external magnetic field $\vec{B}_{ext}$ develops a magnetic dipole moment in the direction of $\vec{B}_{ext}$. If the field is nonuniform, the paramagnetic material is attracted *toward* a region of greater magnetic field *from* a region of lesser field.

The ratio of its magnetic dipole moment to its volume *V*. is the **magnetization M** of the sample, and its magnitude is

$$M = \frac{\text{measured magnetic moment}}{V}.$$

In 1895 Pierre Curie discovered experimentally that the magnetization of a paramagnetic sample is directly proportional to the magnitude of the external magnetic field and inversely proportional to the temperature *T*.

$$M = C\frac{B_{ext}}{T}.$$

is known as *Curie's law,* and *C* is called the *Curie constant.*

Liquid oxygen is suspended between the two pole faces of a magnet because the liquid is paramagnetic and is magnetically attracted to the magnet. *(Richard Megna/Fundamental Photographs)*

32.10: Paramagnetism:

Curie's law is reasonable in that increasing $\boldsymbol{B}_{ext}$ tends to align the atomic dipole moments in a sample and thus to increase M, whereas increasing T tends to disrupt the alignment via thermal agitation and thus to decrease M. However, the law is actually an approximation that is valid only when the ratio B_{ext}/T is not too large.

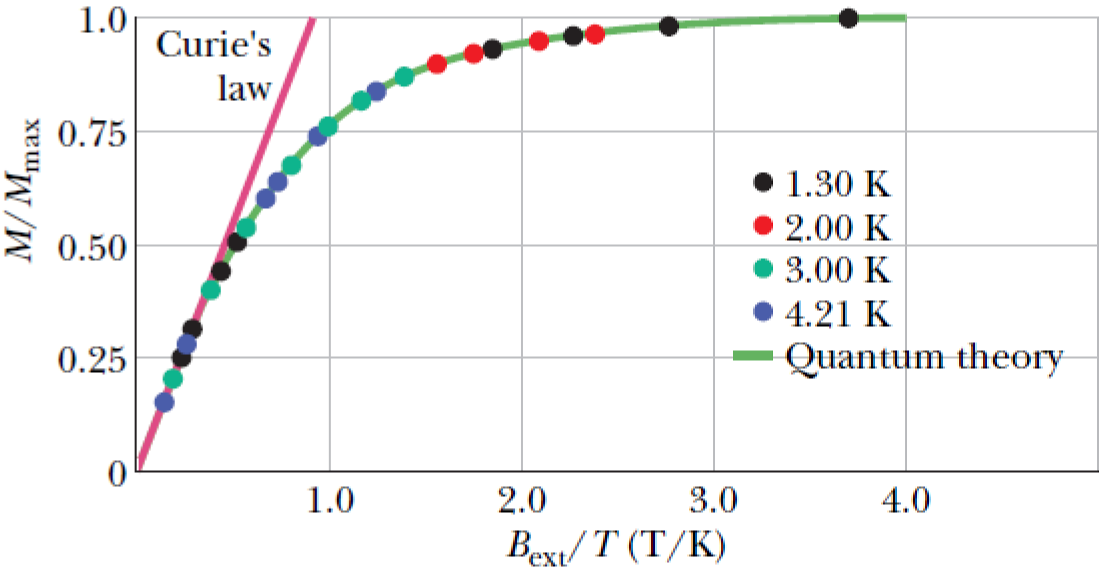


Fig. 32-14 A *magnetization curve* for potassium chromium sulfate, a paramagnetic salt. The ratio of magnetization M of the salt to the maximum possible magnetization M_{max} is plotted versus the ratio of the applied magnetic field magnitude B_{ext} to the temperature T. Curie's law fits the data at the left; quantum theory fits all the data. After W. E. Henry.

Example, Orientation energy of a magnetic field in a paramagnetic gas:

A paramagnetic gas at room temperature ($T = 300$ K) is placed in an external uniform magnetic field of magnitude $B = 1.5$ T; the atoms of the gas have magnetic dipole moment $\mu = 1.0\mu_B$. Calculate the mean translational kinetic energy K of an atom of the gas and the energy difference ΔU_B between parallel alignment and antiparallel alignment of the atom's magnetic dipole moment with the external field.

KEY IDEAS

(1) The mean translational kinetic energy K of an atom in a gas depends on the temperature of the gas. (2) The energy U_B of a magnetic dipole $\vec{\mu}$ in an external magnetic field $\vec{B}$ depends on the angle θ between the directions of $\vec{\mu}$ and $\vec{B}$.

Calculations: From Eq. 19-24, we have

$$K = \tfrac{3}{2}kT = \tfrac{3}{2}(1.38 \times 10^{-23}\ \text{J/K})(300\ \text{K})$$
$$= 6.2 \times 10^{-21}\ \text{J} = 0.039\ \text{eV}. \qquad \text{(Answer)}$$

From Eq. 28-38 ($U_B = -\vec{\mu} \cdot \vec{B}$), we can write the difference ΔU_B between parallel alignment ($\theta = 0°$) and antiparallel alignment ($\theta = 180°$) as

$$\Delta U_B = -\mu B \cos 180° - (-\mu B \cos 0°) = 2\mu B$$
$$= 2\mu_B B = 2(9.27 \times 10^{-24}\ \text{J/T})(1.5\ \text{T})$$
$$= 2.8 \times 10^{-23}\ \text{J} = 0.000\,17\ \text{eV}. \qquad \text{(Answer)}$$

Here K is about 230 times ΔU_B; so energy exchanges among the atoms during their collisions with one another can easily reorient any magnetic dipole moments that might be aligned with the external magnetic field. That is, as soon as a magnetic dipole moment happens to become aligned with the external field, in the dipole's low energy state, chances are very good that a neighboring atom will hit the atom, transferring enough energy to put the dipole in a higher energy state. Thus, the magnetic dipole moment exhibited by the paramagnetic gas must be due to fleeting partial alignments of the atomic dipole moments.

32.11: Ferromagnetism:

A ferromagnetic material placed in an external magnetic field $\vec{B}_{ext}$ develops a strong magnetic dipole moment in the direction of $\vec{B}_{ext}$. If the field is nonuniform, the ferromagnetic material is attracted *toward* a region of greater magnetic field *from* a region of lesser field.

In ferromagnetic materials a quantum physical effect called *exchange coupling* is present where the electron spins of one atom interact with those of neighboring atoms.

The result is alignment of the magnetic dipole moments of the atoms, in spite of the randomizing tendency of atomic collisions due to thermal agitation. This persistent alignment is what gives ferromagnetic materials their permanent magnetism.

If the temperature of a ferromagnetic material is raised above a certain critical value, called the *Curie temperature*, the exchange coupling ceases to be effective. Most such materials then become simply paramagnetic.

32.11: Ferromagnetism:

The magnetization of a ferromagnetic material such as iron can be studied with an arrangement called a *Rowland ring (Fig. 32-15).*

The material is formed into a thin toroidal core of circular cross section. A primary coil P having *n turns* per unit length is wrapped around the core and carries current i_P. If the iron core were not present, the magnitude of the magnetic field inside the coil would be

$$B_0 = \mu_0 i_P n.$$

With the iron core present, the magnetic field inside the coil is greater than $\mathbf{B}_o$, usually by a large amount.

$$B = B_0 + B_M,$$

Here $\boldsymbol{B}_M$ is the magnitude of the magnetic field contributed by the iron core.

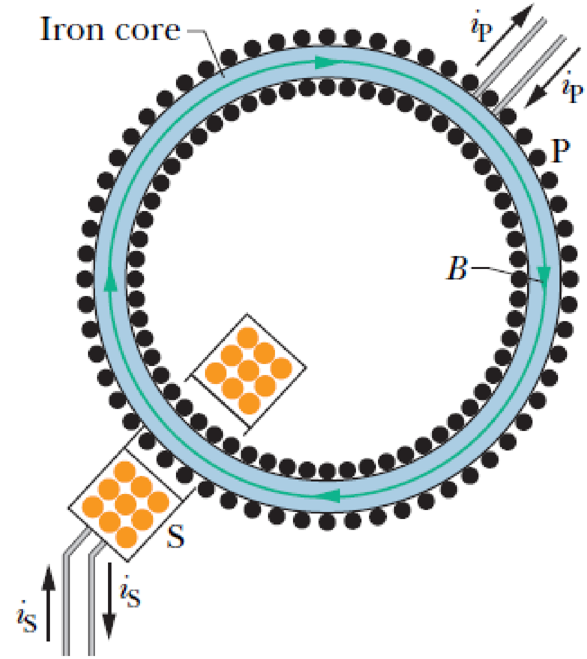


Fig. 32-15 A Rowland ring. A primary coil P has a core made of the ferromagnetic material to be studied (here iron). The core is magnetized by a current i_P sent through coil P. (The turns of the coil are represented by dots.) The extent to which the core is magnetized determines the total magnetic field $\vec{B}$ within coil P. Field $\vec{B}$ can be measured by means of a secondary coil S.

32.11: Ferromagnetism:

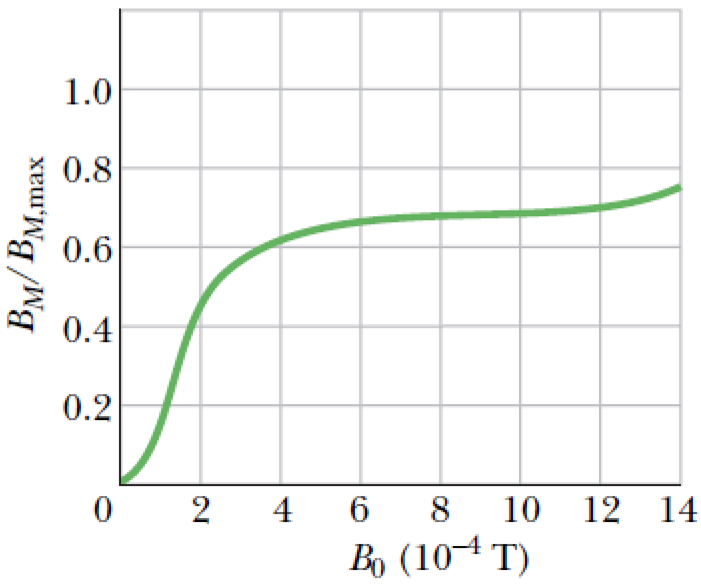


Fig. 32-16 A magnetization curve for a ferromagnetic core material in the Rowland ring of Fig. 32-15. On the vertical axis, 1.0 corresponds to complete alignment (saturation) of the atomic dipoles within the material.

The ratio $B_M/B_{M,max}$, where $B_{M,max}$ is the maximum possible value of B_M, corresponding to saturation, is plotted versus B_0.

The curve is like the magnetization curve for a paramagnetic substance: Both curves show the extent to which an applied magnetic field can align the atomic dipole moments of a material.

For the ferromagnetic core yielding Fig. 32-16, the alignment of the dipole moments is about 70% complete for $B_0 \approx 1 \times 10^{-3}$ *T*.

If B_0 were increased to 1 *T*, the alignment would be almost complete (but B_0 = 1 *T*, and thus almost complete saturation, is quite difficult to obtain).

32.11: Ferromagnetism: Magnetic Domains

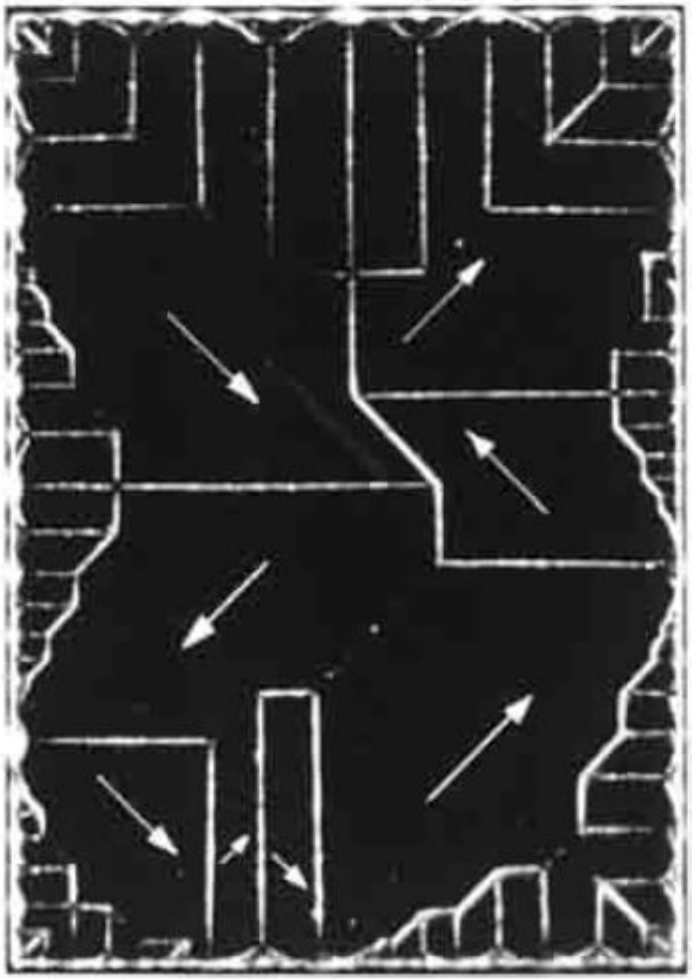


Fig. 32-17 A photograph of domain patterns within a single crystal of nickel; white lines reveal the boundaries of the domains. The white arrows superimposed on the photograph show the orientations of the magnetic dipoles within the domains and thus the orientations of the net magnetic dipoles of the domains. The crystal as a whole is unmagnetized if the net magnetic field (the vector sum over all the domains) is zero. *(Courtesy Ralph W. DeBlois)*

32.11: Ferromagnetism: Hyteresis

Magnetization curves for ferromagnetic materials are not retraced as we increase and then decrease the external magnetic field B_0.

Figure 32-18 is a plot of B_M versus B_0 during the following operations with a Rowland ring:

I. Starting with the iron unmagnetized (point *a*), increase the current in the toroid until B_0 $(=\mu_0 in)$ has the value corresponding to point *b;*
II. reduce the current in the toroid winding (and thus B_0) back to zero (point *c);*
III. reverse the toroid current and increase it in magnitude until B_0 has the value corresponding to point *d;*
IV. reduce the current to zero again (point *e);*
V. reverse the current once more until point *b* is reached again.

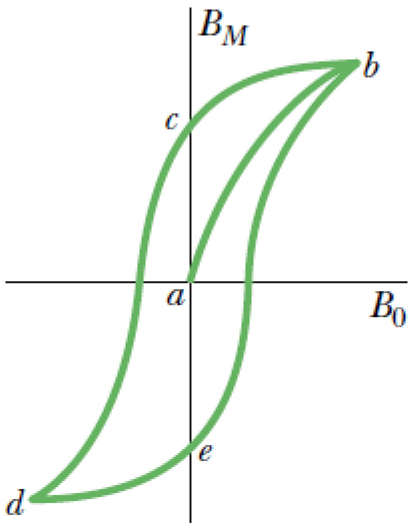


Fig. 32-18 A magnetization curve (*ab*) for a ferromagnetic specimen and an associated hysteresis loop (*bcdeb*).

The lack of retraceability shown in Fig. 32-18 is called **hysteresis**, and the curve *bcdeb* is called a ***hysteresis loop.***

Example, Magnetic Dipole Moment in a Compass Needle:

A compass needle made of pure iron (density 7900 kg/m^3) has a length L of 3.0 cm, a width of 1.0 mm, and a thickness of 0.50 mm. The magnitude of the magnetic dipole moment of an iron atom is $\mu_{Fe} = 2.1 \times 10^{-23}$ J/T. If the magnetization of the needle is equivalent to the alignment of 10% of the atoms in the needle, what is the magnitude of the needle's magnetic dipole moment $\vec{\mu}$?

KEY IDEAS

(1) Alignment of all N atoms in the needle would give a magnitude of $N\mu_{Fe}$ for the needle's magnetic dipole moment $\vec{\mu}$. However, the needle has only 10% alignment (the random orientation of the rest does not give any net contribution to $\vec{\mu}$). Thus,

$$\mu = 0.10 N \mu_{Fe}. \tag{32-42}$$

(2) We can find the number of atoms N in the needle from the needle's mass:

$$N = \frac{\text{needle's mass}}{\text{iron's atomic mass}}. \tag{32-43}$$

Finding N: Iron's atomic mass is not listed in Appendix F, but its molar mass M is. Thus, we write

$$\text{iron's atomic mass} = \frac{\text{iron's molar mass } M}{\text{Avogadro's number } N_A}. \tag{32-44}$$

Next, we can rewrite Eq. 32-43 in terms of the needle's mass m, the molar mass M, and Avogadro's number N_A:

$$N = \frac{mN_A}{M}. \tag{32-45}$$

The needle's mass m is the product of its density and its volume. The volume works out to be 1.5×10^{-8} m^3; so

$$\begin{aligned}\text{needle's mass } m &= (\text{needle's density})(\text{needle's volume}) \\ &= (7900 \text{ kg/m}^3)(1.5 \times 10^{-8} \text{ m}^3) \\ &= 1.185 \times 10^{-4} \text{ kg}.\end{aligned}$$

Substituting into Eq. 32-45 with this value for m, and also 55.847 g/mol (= 0.055 847 kg/mol) for M and 6.02×10^{23} for N_A, we find

$$\begin{aligned}N &= \frac{(1.185 \times 10^{-4} \text{ kg})(6.02 \times 10^{23})}{0.055\,847 \text{ kg/mol}} \\ &= 1.2774 \times 10^{21}.\end{aligned}$$

Finding μ: Substituting our value of N and the value of μ_{Fe} into Eq. 32-42 then yields

$$\begin{aligned}\mu &= (0.10)(1.2774 \times 10^{21})(2.1 \times 10^{-23} \text{ J/T}) \\ &= 2.682 \times 10^{-3} \text{ J/T} \approx 2.7 \times 10^{-3} \text{ J/T}. \qquad \text{(Answer)}\end{aligned}$$