

CHAPTER 23 GAUSS'S LAW PROBLEM SET

- 1) A cube of side l , is placed in a uniform field E_0 with edges parallel to the field lines. (*a*) What is the net flux through the cube? (*b*) What is the flux through each of its six faces?**[Answer: a) Zero, b)** For the faces parallel to **the field lines, flux is zero; For the faces electric field lines entering, flux is** $-El²$ and For the faces electric field lines leaving, flux is $+El²$]
- 2) A uniform field \vec{E} is parallel to the axis of a hollow hemisphere of radius r , Fig. 22–25. (*a*) What is the electric flux through the hemispherical surface? (*b*) What is the result if \vec{E} is instead perpendicular to the axis? $[Answer: a) \pi r^2 E$, b) 0]

3) In Fig. 22–27, two objects, O₁ and O₂, have charges +1.0 × 10⁻⁶ C and -2.0×10^{-6} C respectively, and a third object, O₃, is electrically neutral. (*a*) What is the electric flux through the surface A_1 that encloses all the three objects? (b) What is the electric flux through the surface A_2 that encloses the third object only? **[Answer: a**) -1.1×10^5 *N* \cdot m^2/\mathbf{c} , b) Zero]

4) *** In a certain region of space, the electric field is constant in direction (say horizontal, in the *x* direction), but its magnitude decreases from $E = 560$ N/C at $x = 0$ to $E = 410$ N/C at $x = 25$ m. Determine the charge within a cubical box of side *l=25* m where the box is oriented so that four of its sides are parallel to the field lines (Fig. 22–28).**[Answer:** $Q_{enc} = -8.3 \times 10^{-7} C$]
 $x = 0$ $x = 25$ m

5) *** A nonconducting sphere is made of two layers. The innermost section has a radius of 6.0 cm and a uniform charge density of -5.0 $\frac{C}{m^3}$ The outer layer has a uniform charge density of $+8.0 \, \text{C/m}^3$ and extends from an inner radius of 6.0cm to an outer radius of 12.0cm. Determine the electric field for (*a*) $0 < r < 6.0$ cm, (*b*) 6.0 cm $< r < 12.0$ cm, and (*c*) 12.0 cm $< r < 50.0$ cm. **[Answers: below**

a)
\n**(-1.9×10¹¹ N/C-m)*r*
\n**b)**
\n**c)**
\n
$$
\frac{(-1.1 \times 10^{8} \text{ N} \cdot \text{m}^{2}/\text{C})}{r^{2}} + (3.0 \times 10^{11} \text{ N/C} \cdot \text{m}) r
$$
\n
$$
\frac{(4.1 \times 10^{8} \text{ N} \cdot \text{m}^{2}/\text{C})}{r^{2}}
$$**

6) A flat square sheet of thin aluminum foil, 25 cm on a side, carries a uniformly distributed 275 nC charge. What, approximately, is the electric field (*a*) 1.0 cm above the center of the sheet and (*b*) 15 m above the center of the sheet? [Answer: a) 2.5×10^5 N/C away from the sheet, b)11 N/C away from the **sheet]**

7) *** A very long solid nonconducting cylinder of radius R_1 is uniformly charged with a charge density ρ_E . It is surrounded by a concentric cylindrical tube of inner radius R_2 and outer radius R_3 as shown in Fig. 22–36, and it too carries a uniform charge density ρ_E . Determine the electric field as a function of the distance *R* from the center of the cylinders for (*a*) $0 < R < R_1$, (*b*) $R_1 < R < R_2$, (*c*) $R_2 < R < R_3$, and (*d*) $R > R_3$. **[Answers:**

a) b) c) d)

$$
\frac{\rho_{\rm E}R}{2\varepsilon_0} \qquad \frac{\rho_{\rm E}R_1^2}{2\varepsilon_0 R} \qquad \frac{\rho_{\rm E}\left(R_1^2 + R^2 - R_2^2\right)}{2\varepsilon_0 R} \qquad \frac{\rho_{\rm E}\left(R_1^2 + R_3^2 - R_2^2\right)}{2\varepsilon_0 R}
$$

Fig. 22–36

8) Suppose two thin flat plates measure 1.0 m \times 1.0 m and are separated by 5.0 mm. They are oppositely charged with $\pm 15 \mu C$ (*a*) Estimate the total force exerted by one plate on the other (ignore edge effects). (*b*) How much work would be required to move the plates from 5.0 mm apart to 1.00 cm apart? **[Answer: a)13 N towards the other plate, b)0.064 J]**

9) Three very large sheets are separated by equal distances of 15.0 cm (Fig. 22–47). The first and third sheets are very thin and nonconducting and have charge per unit area **σ** of $+5.00 \mu C/m^2$ and $-5.00 \mu C/m^2$ respectively. The middle sheet is conducting but has no net charge. (*a*) What is the electric field inside the middle sheet? What is the electric field (*b*) between the left and middle sheets, and (*c*) between the middle and right sheets? (*d*) What is the charge density on the surface of the middle sheet facing the left sheet, and (*e*) on the surface facing the right sheet?

[Answer: a) $E = 0$, b) $E = 5.65 \times 10^5$ N/C to the right, c) $E = 5.65 \times 10^5$ 10^5 *N*/*C* to the right, d) $\sigma_{left} = -5.00 \times 10^{-6}$ *C*/*m*², e) $\sigma_{right} =$ $+5.00 \times 10^{-6}$ C/m²]

10) A hemisphere of radius *R* is placed in a charge-free region of space where a uniform electric field exists of magnitude *E* directed perpendicular to the hemisphere's circular base (Fig. 22–50). (*a*) Using the definition of ϕ_E through an "open" surface, calculate (via explicit integration) the electric flux through the hemisphere. [*Hint*: In Fig. 22–50 you can see that, on the surface of a sphere, the infinitesimal area located between the angles θ and θ +d θ is

 $dA = (2\pi R \sin\theta)(R d\theta) = 2\pi R^2 \sin\theta d\theta$ (*b*) Choose an appropriate gaussian surface and use Gauss's law to much more easily obtain the same result for the electric flux through the hemisphere.

[Answer: a) $\pi r^2 E$, b) $\pi r^2 E$]