

#### CHAPTER 23 GAUSS'S LAW PROBLEM SET

- 1) A cube of side l, is placed in a uniform field  $E_0$  with edges parallel to the field lines. (a) What is the net flux through the cube? (b) What is the flux through each of its six faces?[Answer: a) Zero, b) For the faces parallel to the field lines, flux is zero; For the faces electric field lines entering, flux is  $-El^2$  and For the faces electric field lines leaving, flux is  $+El^2$ ]
- 2) A uniform field Ē is parallel to the axis of a hollow hemisphere of radius r, Fig. 22–25. (a) What is the electric flux through the hemispherical surface? (b) What is the result if Ē is instead perpendicular to the axis?
  [Answer: a)πr<sup>2</sup>E, b) 0]



3) In Fig. 22–27, two objects,  $O_1$  and  $O_2$ , have charges  $\pm 1.0 \times 10^{-6} C$  and  $-2.0 \times 10^{-6} C$  respectively, and a third object,  $O_3$ , is electrically neutral. (a) What is the electric flux through the surface  $A_1$  that encloses all the three objects? (b) What is the electric flux through the surface  $A_2$  that encloses the third object only? [Answer: a)  $-1.1 \times 10^5 N \cdot \frac{m^2}{C}$ , b) Zero]





4) \*\*\* In a certain region of space, the electric field is constant in direction (say horizontal, in the x direction), but its magnitude decreases from E = 560 N/C at x = 0 to E = 410 N/C at x = 25 m. Determine the charge within a cubical box of side *l=25* m where the box is oriented so that four of its sides are parallel to the field lines (Fig. 22–28).[Answer:  $Q_{enc} = -8.3 \times 10^{-7} C$ ]



5) \*\*\* A nonconducting sphere is made of two layers. The innermost section has a radius of 6.0 cm and a uniform charge density of -5.0 C/m<sup>3</sup> The outer layer has a uniform charge density of +8.0 C/m<sup>3</sup> and extends from an inner radius of 6.0 cm to an outer radius of 12.0 cm. Determine the electric field for (a) 0 < r < 6.0 cm, (b) 6.0 cm < r < 12.0 cm, and (c) 12.0 cm < r < 50.0 cm. [Answers: below]</li>

a)  

$$\begin{pmatrix} -1.9 \times 10^{11} \text{ N/C-m} \end{pmatrix} r$$
b)  
c)  

$$\frac{(-1.1 \times 10^{8} \text{ N-m}^{2}/\text{C})}{r^{2}} + (3.0 \times 10^{11} \text{ N/C-m}) r$$

$$\frac{(4.1 \times 10^{8} \text{ N-m}^{2}/\text{C})}{r^{2}}$$
]

6) A flat square sheet of thin aluminum foil, 25 cm on a side, carries a uniformly distributed 275 nC charge. What, approximately, is the electric field (a) 1.0 cm above the center of the sheet and (b) 15 m above the center of the sheet?
[Answer: a)2. 5 × 10<sup>5</sup> N/C away from the sheet, b)11 N/C away from the sheet]



7) \*\*\* A very long solid nonconducting cylinder of radius  $R_1$  is uniformly charged with a charge density  $\rho_E$ . It is surrounded by a concentric cylindrical tube of inner radius  $R_2$  and outer radius  $R_3$  as shown in Fig. 22–36, and it too carries a uniform charge density  $\rho_E$ . Determine the electric field as a function of the distance R from the center of the cylinders for (*a*)  $0 < R < R_1$ , (*b*)  $R_1 < R < R_2$ , (*c*)  $R_2 < R < R_3$ , and (*d*)  $R > R_3$ . [Answers:

a) b) c) d)  

$$\frac{\rho_{\rm E}R}{2\varepsilon_0} = \frac{\rho_{\rm E}R_1^2}{2\varepsilon_0 R} = \frac{\rho_{\rm E}\left(R_1^2 + R^2 - R_2^2\right)}{2\varepsilon_0 R} = \frac{\rho_{\rm E}\left(R_1^2 + R_3^2 - R_2^2\right)}{2\varepsilon_0 R}$$



Fig. 22–36

8) Suppose two thin flat plates measure 1.0 m × 1.0 m and are separated by 5.0 mm. They are oppositely charged with ±15 μC (a) Estimate the total force exerted by one plate on the other (ignore edge effects). (b) How much work would be required to move the plates from 5.0 mm apart to 1.00 cm apart? [Answer: a)13 N towards the other plate, b)0.064 J ]



9) Three very large sheets are separated by equal distances of 15.0 cm (Fig. 22-47). The first and third sheets are very thin and nonconducting and have charge per unit area σ of +5.00 µC/m<sup>2</sup> and -5.00 µC/m<sup>2</sup> respectively. The middle sheet is conducting but has no net charge. (a) What is the electric field inside the middle sheet? What is the electric field (b) between the left and middle sheets, and (c) between the middle and right sheets? (d) What is the charge density on the surface of the middle sheet?

[Answer: a) E = 0, b) $E = 5.65 \times 10^5 N/C$  to the right, c)  $E = 5.65 \times 10^5 N/C$  to the right, d) $\sigma_{left} = -5.00 \times 10^{-6} C/m^2$ , e)  $\sigma_{right} = +5.00 \times 10^{-6} C/m^2$ ]



**10)** A hemisphere of radius *R* is placed in a charge-free region of space where a uniform electric field exists of magnitude *E* directed perpendicular to the hemisphere's circular base (Fig. 22–50). (*a*) Using the definition of  $\phi_E$  through an "open" surface, calculate (via explicit integration) the electric flux through the hemisphere. [*Hint*: In Fig. 22–50 you can see that, on the surface of a sphere, the infinitesimal area located between the angles  $\theta$  and  $\theta$ +d $\theta$  is

 $dA = (2\pi R sin\theta)(Rd\theta) = 2\pi R^2 sin\theta d\theta](b)$  Choose an appropriate gaussian surface and use Gauss's law to much more easily obtain the same result for the electric flux through the hemisphere.

[Answer: a)  $\pi r^2 E$ , b)  $\pi r^2 E$ ]